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EXPERIMENTS IN ULTRA-VIOLET REFRACTOMETRY

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ABSTRACT. The experiments described have for their object the application of critical angle methods for the refractometry of liquids in the ultra-violet. A thin film of liquid can be held between two quartz hemispheres which are traversed centrally by an approximately parallel beam; the film receives the radiation at the varying angles of incidence resulting on rotation of the system. In this way, analysing the transmitted radiation with the aid of a quartz spectrograph, the critical angles for definite wave-lengths are measured, from which refractive indices can be calculated. The procedure necessary in seeking precise results is discussed, and a series of measurements on glycerine-water mixtures likely to be useful for immersion fluids in ultra-violet microscopy is given. A set of interesting phenomena of the extinction bands is described and explained.

INTRODUCTION

THE renewed attention which is being given to ultra-violet microscopy has emphasised the necessity for the measurement of refractive indices of liquids for ultra-violet radiation. It is usually necessary to provide an "immersion fluid" of some stated refractive index for a given wave-length; and moreover in bacteriological work it is most desirable to be able to measure, or check, the constancy of the refractive index of the culture medium. For purposes such as the latter, only small quantities of the medium will usually be available.

REQUIRED ACCURACY

The thickness of the film of immersion fluid will be of the order of 0.1 mm.; that of the cover slip may be 0.2 mm.; we may take an example in which the object

point is at a depth of 0.01 mm. below the surface of the slip. For a lens of real N.A. 1.3 the divergence angle α , Fig. 1, between the extreme ray and the axis in the cover slip, immersion fluid, and front lens will be the same if these (as we will assume) have the same refractive indices for the ultra-violet radiation; we assume the wavelength 0.275μ and the refractive index of the media (that of fused quartz) as 1.496. Then

$$\text{N.A.} = 1.3 = 1.496 \sin \alpha,$$

whence we find that α is approximately 60° . If N_m is the refractive index of the medium in which the object is situated, and α' the divergence angle here, then

$$N_m \sin \alpha' = \text{N.A.} = 1.3.$$

Assuming that N_m is about 1.37 for this wave-length (a not unlikely value), we find α' to be just over 71° .

The difference of path of the axial and extreme rays in the object medium then amounts to $0.01 (\sec 71^\circ - 1)$ mm., which comes to about 0.02 mm. If the lens is working satisfactorily under this condition, let a change of refractive index δN of the culture medium take place; then the difference of optical path so introduced between the axial and marginal rays is $\delta N \times 0.02$; if this exceeds $\lambda/4$ a change of focus will be required. Hence $\delta N = \frac{0.275 \times 10^{-3}}{4 \times 0.02} = 0.0034$ for the maximum permissible variation of refractive index.

Again the difference of path in the immersion fluid amounts to $0.1 (\sec 60^\circ - 1) = 0.1$ mm. The change of refractive index just sufficient to cause a change of focus then proves to be 0.0007. These figures are only representative, of course, in work of the most extreme precision and accuracy, but it will be evident that any adequate refractive method should be capable of giving results, say, to within four or five units of the fourth decimal place of refractive index, although results correct even to the third decimal place should be very useful in many cases. No satisfactory apparatus for the ultra-violet refractometry of very small quantities of liquid appears so far to have been brought out, although Messrs Adam Hilger provide an apparatus for use when sufficient of the substance is available to fill a small wedge cell.

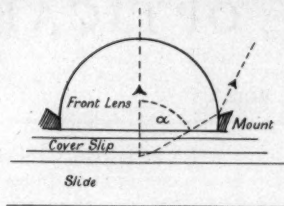


Fig. 1.

BASIC IDEA

It was decided that a trial might be made of a critical angle method, and the first simple optical idea which suggested itself is shown in Fig. 2. Radiation is focussed in slit 1, whence a narrow divergent beam proceeds to the surface of a sphere cut from quartz so that the axis is perpendicular to the plane of the diagram. This sphere is divided across its diameter for the introduction of a thin film of liquid; it can rotate about the diameter also perpendicular to the diagram and

parallel to the crystal axis. The slits are situated at the principal foci of the two opposite surfaces so that a parallel beam passes through the sphere, and the image

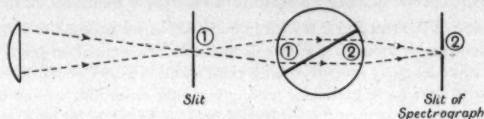


Fig. 2.

of slit 1 is coincident with slit 2 which is the slit of a spectrograph adjusted to register the spectrum of the transmitted energy.

When the film is normal to the incident radiation the spectrum will be complete, but when it is turned so that the angle of incidence on the film is i , where

$$N_q \sin i = N_l \quad \dots (1)$$

(N_q and N_l being the refractive indices of quartz and liquid respectively for some given wave-length), total reflection will occur and the corresponding radiation will not be present in the spectrum. Owing to the differences of dispersion generally to be expected, the extinction will be complete, usually, on one side of the critical wave-length; as the angle of incidence is varied, the extinction will progress through the spectrum.

It is necessary, of course, to consider the action of the above system much more closely, and this will be done below.

CONDITIONS FOR EXTINCTION

In order to understand the various cases that may be encountered, it is helpful to consider Fig. 3 in which refractive indices are plotted as ordinates against wave-lengths as abscissae. The curves for the ordinary and extraordinary refractive indices of quartz are shown and also those for the imaginary liquids a and b . The ordinates of the dotted curves intersecting the a and b curves are proportional to those of the ordinary ray.

The condition for the extinction of radiation of a given wave-length is given in equation (1) above. Regarding $\sin i$ as the factor by which the quartz index should be multiplied to make it equal to N_l it will be seen that for any case $\sin i$ must be smaller for the extraordinary than the ordinary ray. For any greater angle of incidence the extraordinary ray will be totally reflected; hence, as the angle of incidence increases, the ordinary ray is always the last to disappear.

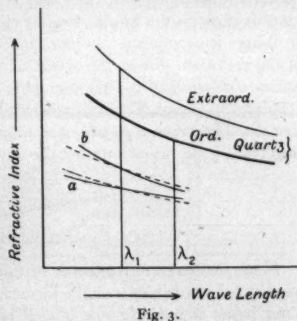


Fig. 3.

Now take curve *a* and consider the action at wave-lengths λ_1 and λ_2 . If

$$N_q k = N_i,$$

then the value of the fraction *k* evidently increases as we proceed to longer wave-lengths. Hence with increasing angles of incidence the extinction point travels to the longer wave-lengths; moreover since the condition $N_q k' > N_i$, where *k'* may now represent $\sin i$ for a particular case, gives total reflection, we see that if extinction just occurs at λ_1 then total reflection will be complete for all shorter wave-lengths.

Curve *b* shows the opposite condition; with increasing angles of incidence the extinction point travels to the shorter wave-lengths, while the reflection is complete for all longer wave-lengths than the extinction point.

From the extinction relation $k = N_i/N_q$ we obtain

$$\frac{dk}{d\lambda} = \frac{\frac{dN_i}{d\lambda} N_q - \frac{dN_q}{d\lambda} N_i}{N_q^2} \quad \dots (2),$$

from which the sign of the extinction direction can be inferred. A particular case of interest is given when the numerator on the right is zero, or when

$$\frac{dN_q/d\lambda}{dN_i/d\lambda} = \frac{N_q}{N_i},$$

which would be graphically represented in Fig. 3 by the osculation of the dotted curve and "liquid" curve. The meaning of this is seen from the infinite value of $d\lambda/dk$; the extinction is simultaneous for the whole spectral region within which the above relation is true. Owing to the similar nature of the dispersion curves of quartz and some liquids, cases may arise in which the extinction is almost simultaneous through the whole ultra-violet spectrum.

There is naturally a "border line" characterised by the increase in brightness of the spectrum due to the action of the extraordinary beam. If this sometimes causes confusion in the photographic record it is possible to avoid all such uncertainty by using a Foucault polarising prism in the path of the light entering the first slit. The prism is so placed as to give "horizontal vibrations" which will form the ordinary ray when entering the quartz with its vertical crystal axis.

OPTICAL ACTION OF THE SYSTEM

Since temperature control is of the greatest importance, the "prism" system constructed in practice took the form shown in section and elevation in Fig. 4. The water jacket is made in two hinged parts and these hold the prism components together or permit them to be separated when unclamped. The whole was made with great care and skill by Messrs Bellingham and Stanley, Ltd., but it would not

be safe to assume the exact parallelism of the faces enclosing the film, or that faces 1 and 2 (Fig. 2) lie in exactly the same spherical locus.

Under the conditions it is possible to use the system in two distinct ways.

(1) Mount the system so that the centre of rotation passes through the centre of curvature of face 1. Make slit 1 fairly narrow, and examine the corresponding image with a spectrograph.

(2) Use the first component of the system simply as an illuminating system adapted, as in the Abbe refractometer, to bring radiation at grazing incidence on to the flat surface of the second component. In the simple form of the apparatus, the rotation should then be strictly about the centre of curvature of face 2. The first slit will be wide and is only used to stop an excess of light and consequent back reflections, etc.; the slit of the spectrograph is very narrow.

A certain number of preliminary experiments were made by method (1), but more attention has been given to a modification of method (2) by which it is probably possible to obtain more exact results. Nevertheless the first method presents some features of interest, and it may be examined in more detail, but first some general experimental points may be discussed. The source of light for the first experiment was a small iron arc; a $1\frac{1}{2}$ inch quartz condenser forms an image of the arc on slit 1 at a distance of 24 inches; this determines the semi-angular aperture of the beam falling on face 1, which is less than 2° . The iron arc is employed for its wealth of lines in the ultra-violet spectrum, which permits the estimation of extinction points, but it can be improved upon by the use of a tungsten arc or the condensed aluminium spark under water.

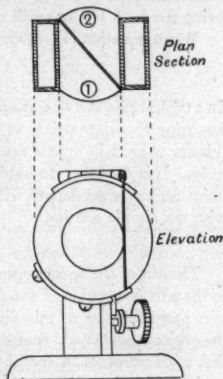


Fig. 4. The "prisms" and water jacket. (The connecting tubes for the water circulation are not shown.)

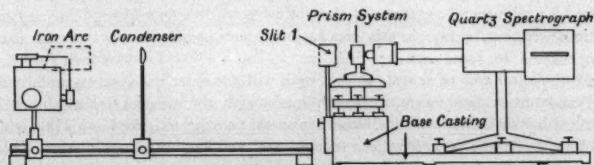


Fig. 5. General arrangement of apparatus.

The radius of curvature of the quartz surface is 0.54 inch, whence the focal distance for a refractive index of 1.56 ($\lambda = 0.38\mu$ approx.) is 0.96 inch, and the slit 1 is accordingly placed at this distance from the surface. The "prism system" is

mounted, as seen in Fig. 5, on a theodolite table having a divided circle reading by vernier to 10 seconds of arc. Readings were taken on one vernier by a low power microscope, the eccentricity corrections having been found to be of the order of 5 seconds only. Corrections for vernier length were duly applied. The theodolite table stands on a casting which also supports the spectrograph, the bases of both being fixed. A Hilger small quartz spectrograph was used.

When equation (1) above is differentiated we obtain

$$dN_1 = N_1 \cos i \, di \quad \dots (3).$$

In a typical case at $\lambda = 0.412 \mu$, $N_1 = 1.5562$, $i = 61^\circ 30'$ (approx.) and $N_2 = 1.3675$. Inserting the appropriate values we find that an error of estimation of i by one minute of arc will in this case cause an error of 0.0002 in the calculated refractive index. It is therefore desirable that we should be able to fix the angular extinction point for any wave-length within this limit. The accuracy increases in accordance with the simple relation

$$dN_1/N_1 = \cot i \, di \quad \dots (4).$$

The slit of the spectrograph was used in a direction parallel to the first slit and to the axis of rotation of the prism. It would be possible, of course, to place both slits perpendicular to this direction, when the extinction band corresponding to the critical angle would evidently run diagonally through the spectrum. It is evident that measurements of the plate would be involved in the work, or some special device adopted, and the first plan avoids this.

METHOD (1)

In this method one seeks to observe the extinction of rays of any one wave-length by total reflection, using prism 2 and the spectrograph as a means of collecting and observing the radiation *not* reflected in prism 1. It would take too long to discuss this properly, but angular measurements may be subject to errors of the order of 4 minutes of arc through chromatic aberration, although this can be compensated to some extent; thus in Fig. 6 the outer rays of shorter wave-length will not enter the spectrograph.

Some observations were, however, made with the simple system shown in Fig. 2, using slits about 0.1 mm. broad; as would be expected, the border line is not absolute, but there is a definitely marked and sudden brightness change which represents the border, and this sweeps through the spectrum as the angle of incidence is changed.

Some measurements of the refractive index of water made by this method will be found below.

It would clearly be possible to achromatise the refraction at the first face, but

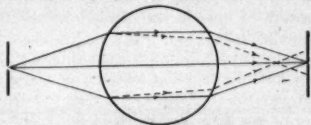


Fig. 6. Effect of chromatic aberration.

it will be seen that this procedure would have no advantage over that adopted in Method (2) unless a radically different method of detecting the transmitted radiation is employed such as is suggested in Fig. 7. A fluor-quartz objective parallelises

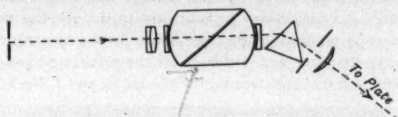


Fig. 7. Achromatised system for Method (1).

the incident beam, which is transmitted into the prism system by a plano-concave lens of quartz; a film of glycerine could be used between lens and "prism." On the other side is found a dispersion system analogous to that used in a quartz spectrograph of ordinary design. The results of the present work indicate that such an instrument, if properly made, could be used for obtaining results of precision, but there has been no opportunity, at present, to try such a design.

METHOD (2)

This method is probably by far the most suitable for the apparatus at present available, and the complete optical system actually tested is shown in Fig. 8.

The use of a negative lens (n) and achromatic objective (a) follows the con-

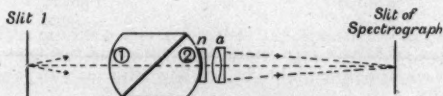


Fig. 8. System used in Method (2).

struction of the crystal refractometer of Messrs Zeiss, although in the present case the border-line image falls on the slit of the spectrograph (made as narrow as possible consistently with obtaining sufficiently short exposures). Slit 1 is 2 mm. broad, and its twice magnified image projected by the optical system falls over the spectrograph slit.

In this case the "slit" and prism 1 may be regarded as an illuminating system which brings light into the film at grazing incidence (and at a range of lesser angles) on the surface of prism 2. For each wave-length there is then a border line the image of which lies in the focal surface of the objective a and sweeps over the slit as the film reaches the critical angular position.

The concave surface of n is very close to prism 2, and is of the same curvature. A thin film of glycerine minimises any error arising from a departure of the moving surface from the proper spherical locus, or slight relative movements of 2 and n .

It is evidently of great importance that the collimation line determined by the

objective a and the spectrograph slit shall not rotate with respect to the vertical axis of rotation of the prisms. The system of n and a is held in a mount clamped over the slit fitting of the spectrograph as shown in Fig. 5. All parts including the spectrograph are mounted on a specially made casting. The lens a can be moved for focussing. Naturally it is necessary to take care to disturb the parts as little as possible when inserting or moving the plate holder in the spectrograph.

A fluorite-quartz objective was designed for the present purpose by the writer to use existing tools, and was constructed by Messrs R. and J. Beck, Ltd. The data are:

$$\begin{array}{l} \text{Fluorite lens} \quad \left\{ \begin{array}{l} r_1 = 0.417 \quad d = 0.07 \\ r_2 = -0.485 \end{array} \right. \\ \text{Quartz lens} \quad \left\{ \begin{array}{l} r_3 = -0.485 \quad d = 0.04 \\ r_4 = 1.4204 \end{array} \right. \end{array} \quad \text{Diameter } 0.24$$

All dimensions in inches.

The quartz lens to be cut with its axis perpendicular to the crystal axis.

The axis direction to be marked.

The lens is sufficiently spherically corrected for $\lambda = 0.36\mu$ and has a minimum focus for $\lambda = 0.35\mu$ approximately, the intersection distances L_m for the marginal ray being

λ (μ)	L_m (in.)
0.2313	1.97623
0.36107	1.96523
0.4046	1.96788

The corrections could have been improved by the use of new tools, but the performance was amply sufficient with the lens as constructed with tools which were already available.

It will be realised that the prism system acts in the neighbourhood of the critical angle very similarly to an ordinary polariser or analyser; the ordinary vibrations (horizontal) will alone be transmitted. The quartz component of the objective therefore is mounted with its crystal axis vertical, and the same applies to the lens n . Thus all trouble due to double images is avoided and the ordinary ray need alone be considered. The objective is mounted in a focussing slide and is so placed that the spectrograph slit is in its focal plane.

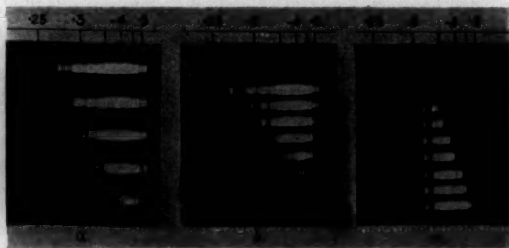
METHOD OF USE

The refractive indices of quartz have been accurately measured for a range of wave-lengths in the visible and ultra-violet regions of the spectrum. Hence the extinction positions for known wave-lengths when an air film is between the prisms give a measure of the angles of incidence for the radiation. For present purposes a large-scale curve was drawn from which the refractive index of quartz* for a given wave-length could be read to within two units of the fourth decimal place. Pre-

* From Gifford's tables, *Proc. Roy. Soc.* 70 1902 (329); 84 1919 (193).

viously to commencing the work, a photograph of the iron spectrum was taken on the spectrograph, and a wave-length scale added by Messrs Hilger's convenient arrangement. A large-scale map of the iron spectrum then permits the identification of individual lines, and a correction curve for the wave-length scale can be drawn. In this way it is easy to estimate the extinction positions with ample accuracy.

In one of the earlier experiments the plate used was an Ilford "Iso-Zenith." The spectrum was first observed visually and the angle of incidence on the film



(a) Typical air plate, angular interval = $15'$. (b) Typical water plate, angular interval = $3'$. (c) Castor oil: showing reversed extinction direction due to higher relative dispersion. No transmission takes place for shorter wave-lengths than about 0.292μ . Angular interval = $7' 30''$.

Fig. 9.

increased till the extinction appeared in the visible spectrum. The plate was given a series of 2 minutes' exposures, the theodolite table being turned, between exposures, through an angle of 15 minutes of arc registered on the vernier. Fig. 9, *a* shows the appearance of a typical air plate. The results shown in Table I were obtained.

Table I

Circle reading	Wave-length for extinction	Refractive index of quartz	Calculated angle of incidence	Correction to circle reading to give angle of incidence*
	μ			
$14^{\circ} 15'$	0.396	1.5582	$39^{\circ} 55' 23''$	$25^{\circ} 40' 23''$
0	0.350	1.5656	$39^{\circ} 41' 50''$	$25^{\circ} 41' 50''$
$13^{\circ} 45'$	0.316	1.5734	$39^{\circ} 27' 43''$	$25^{\circ} 42' 43''$
30	0.291	1.5812	$39^{\circ} 13' 47''$	$25^{\circ} 43' 47''$
15	0.271	1.5893	$38^{\circ} 59' 30''$	$25^{\circ} 44' 30''$

* When the circle is "set" to definite angular vernier readings with the aid of the microscope, the settings will probably be accurate within about 5 seconds of arc. It is frequently the case in refractometry that angular differences must be measured accurately to get the best results for dispersion, even when the absolute accuracy is not correspondingly good. Hence numerical results are calculated to one second.

The figures in the last column on the right should, of course, agree. The precise reason for the progressive error was not at first clear since a variable prismatic action between 2 and n (Fig. 8), owing to imperfect adjustment of the centre of rotation, changes of temperature, strong light and a number of other uncertainties, was considered. The trouble was shown by a series of experiments to be caused by using too wide a spectrograph slit, which was of the order of 0.2 mm. By reducing the slit to 0.01 mm. approximately, the angular subtense of the breadth at the focal distance of the objective is reduced to less than 1 minute of arc.

A new plate taken with a fine slit now gave the results shown in Table II (the adjustment had been somewhat altered).

Table II

Circle reading	Calculated correction (as above)
14° 15'	25° 42' 5"
0	25 41 7
13 45	25 42 54
30	25 43 15
15	25 42 55
	Mean 25° 42' 25"

The length of exposure thus required to give reasonable rendering in the shorter wave-lengths is about 3 minutes. The results seemed to be reasonably satisfactory, and justified the taking of a mean value for the calculated "air plate correction" which must be added to the circle reading to obtain the angle of incidence on the film. It is naturally desirable to take an "air plate" to accompany any other measurements in order to check the constancy of the adjustment.

RESULTS BY METHOD (1)

Naturally a good many trials were made. Water is one of the most useful media since its ultra-violet refractive indices are fairly accurately known from the work of Flatow*. In one of the early trials with comparatively wide slits it was found that series of exposures with angles of incidence differing by 3 minutes of arc gave a satisfactory series of extinction positions throughout the spectrum. Applying the mean zero correction found from the corresponding air plate, the results shown in Table III were obtained (laboratory temperature).

Table III

Wave-length	Experimental results	Flatow's results (interpolated from curve)	Error
μ			
0.400	1.3439	1.3430	+0.0009
0.365	1.3479	1.3471	0.0008
0.345	1.3502	1.3498	0.0004
0.315	1.3556	1.3548	0.0008
0.291	1.3614	1.3608	0.0006
0.270	1.3684	1.3679	0.0005
0.249	1.3771	1.3770	0.0001

* *Ann. d. Phys.* (4), 12 1903 (85); Landolt and Bornstein, *Phys. Chem. Tabellen*, 1923.

Considering the simplicity of the apparatus the results are not unsatisfactory; there is some evidence of an accidental error in the third result (for $\lambda = 0.345$) but the other errors are apparently systematic and due to the breadth of the slits. The above results are typical of a good many trials, but the presence of a systematic error of the above kind cannot be tolerated if accuracy is required, because its amount would be most difficult to predict in other cases. Hence Method (2) was used for the more careful trials.

RESULTS BY METHOD (2)

Table IV illustrates the computation of results obtained from an experiment on distilled water at 16°C. , the temperature being controlled by water circulation around the prisms. The manner of computing the results will be understood.

Table IV. *Distilled Water at 16°C.*

Circle reading	Vernier correction	Angle of incidence	Wave-length for extinction (μ)	N_g from curve	$\log N_g + \log \sin i$	N_i experiment	Flatow's results (interpolated)	Error
$33^\circ 57'$	$1' 2''$	$59^\circ 40' 32''$	0.402	1.5873	0.12847	1.3442	1.3433	0.0009
$33^\circ 54'$	0.47	$59^\circ 37' 17''$	0.380	1.5603	0.12907	1.3461	1.3456	0.0005
$33^\circ 51'$	0.32	$59^\circ 34' 2''$	0.345	1.5666	0.13058	1.3508	1.3501	0.0007
$33^\circ 48'$	0.16	$59^\circ 30' 46''$	0.310	1.5750	0.13266	1.3572	1.3563	0.0009
$33^\circ 45'$	0.0	$59^\circ 27' 30''$	0.289	1.5818	0.13429	1.3623	1.3615	0.0008
$33^\circ 42'$	1.2	$59^\circ 25' 32''$	0.260	1.5947	0.13767	1.3730	1.3725	0.0005

The residual error determined by the experiment described and by several others on water is about 7 units in the fourth decimal place, but it is reasonably constant within the expected limits of error (the accidental errors are mainly incurred through possible errors in reading off the refractive index of quartz from the curve, failure to locate correctly the wave-length of extinction, etc.). In order to estimate the error δN_i due to an error $\delta \lambda$ in the latter estimation we may write from equation (1),

$$N_i + \delta \lambda \cdot dN_i/d\lambda + \delta N_i = (N_g + \delta \lambda \cdot dN_g/d\lambda) \sin i,$$

whence

$$\delta N_i = \delta \lambda (\sin i \cdot dN_g/d\lambda - dN_i/d\lambda) \quad \dots (5).$$

As was obvious from equation (2), when $\sin i = \frac{dN_i}{d\lambda} / \frac{dN_g}{d\lambda}$, the error in the estimation of the extinction point has no influence on the results, and in fact the estimation need never be one of great accuracy when a liquid is being tested. Numerical computation shows, in fact, that in measurements on water an error of, say, 0.01μ in estimating the extinction point will not cause errors which are appreciable with regard to the other errors. The extinction border travels from 0.25 to 0.400μ in an angular movement of 18 minutes of arc in the case of water; hence 0.01μ corresponds roughly to 1 minute, and an uncertainty of about 2 units in the fourth decimal place of refractive index. The plates taken on the spectrograph seemed, however, as if they would allow the fixing of the extinction position to within 0.005μ .

The above remarks do not apply to the case of the "air plate"; the estimation of the extinction point is apparently possible to within 0.002μ , but the phenomenon of total reflection is not characterised by a *total* discontinuity, nor is the extinction in the spectrum absolutely sharp. The estimated wave-length of extinction on an "air plate" may easily be somewhat high (as the plate fails to register up to the absolute limit) and this is the most probable cause of the high values obtained for water. We are therefore justified in applying a correction to the angle added to bring the circle readings up to angles of incidence. The required correction for the "water" table will be obtained by reducing the estimated angle of incidence by about 3 minutes. The corrected results for water are given in Table V.

Table V

Wave-length	Refractive index	Flatow's results	Error
μ			
0.402	1.3435	1.3433	+0.0002
0.380	1.3454	1.3456	-0.0002
0.345	1.3501	1.3501	+0.0000
0.310	1.3565	1.3563	+0.0002
0.280	1.3626	1.3615	+0.0001
0.260	1.3723	1.3725	+0.0002

The result for 0.345 is now correct, but there is still a residual error greatest in the shorter wave-lengths although its magnitude is below the probable error of experiment and will not therefore receive further discussion.

This correction is made to all subsequent results.

It may be noted that the film was fine enough to exhibit interference effects (see below) and that small departures from absolutely grazing incidence on the standard block of the prism do not appreciably affect the angle of refraction. If α is the angle of incidence in the film,

$$N_i \sin \alpha = N_a \sin i$$

and

$$N_i \cos \alpha \, da = N_a \cos i \, di$$

whence

$$di = da (N_i \cos \alpha) / (N_a \cos i).$$

Since at angles near grazing incidence $\cos \alpha$ is very small, the change in i due to a small change in α will be small of the second order. This is very important with regard to all refractometers depending on grazing incidence.

In order to give the apparatus a trial it was decided to make a series of measurements on glycerine-water mixtures which are likely to be useful in ultra-violet microscopy. Some of the mixtures are strongly hygroscopic, while others will change through evaporation; precautions were taken not to expose them to the air. When the liquid is pressed into a film in the refractometer, the refractive index changes only very slowly; a separate experiment showed the change in the refractive index of mixture D to be less than one unit in the fourth place per hour under these conditions. In addition to the ultra-violet experiments, measurements were also

made for the usual hydrogen and sodium lines of the visible spectrum, using a refractometer of the Pulfrich type. In all cases the temperature was controlled as carefully as possible.

Table VI. Mixture A. Percentage of water by weight = 79.96

Visible spectrum (20° C.)		Ultra-violet (20-20.1° C.)	
λ (μ)	N	λ (μ)	N
0.6563 (C)	1.3557	0.412	1.3668
0.5893 (D)	1.3576	0.371	1.3712
0.4861 (F)	1.3620	0.344	1.3751
0.4341 (G)	1.3653	0.311	1.3818
—	—	0.297	1.3851
—	—	0.260	1.3983
—	—	0.234	1.4123

Mixture B. Percentage of water = 60.05

Visible (20° C.)		Ultra-violet (20.3-20.5° C.)	
λ (μ)	N	λ (μ)	N
0.6563	1.38161	0.358	1.4017
0.5893	1.38356	0.331	1.4063
0.4861	1.38829	0.310	1.4112
0.4341	1.39190	0.275	1.4220
—	—	0.250	1.4337

Mixture C. Percentage of water = 40.12

Visible (21° C.)		Ultra-violet (20° C.)	
λ (μ)	N	λ (μ)	N
0.6563	1.4097	0.40	1.4238
0.5893	1.4119	0.36	1.4294
0.4861	1.4168	0.32	1.4372
0.4341	1.4206	0.28	1.4492
—	—	0.24	1.4688

Mixture D. Percentage of water = 20.56

Visible (20° C.)		Ultra-violet (20° C.)	
λ (μ)	N	λ (μ)	N
0.6563	1.4385	0.40	1.4549
0.5893	1.4405	0.36	1.4606
0.4861	1.4460	0.32	1.4686
0.4341	1.4501	0.28	1.4808
—	—	0.24	1.5008

L. C. Martin
Commercial Glycerine

Visible (20° C.)		Ultra-violet (20° C.)	
λ (μ)	N	λ (μ)	N
0.6563	1.4698	0.388	1.4873
0.5893	1.4721	0.322	1.5003
0.4861	1.4776	0.284	1.5128
0.4341	1.4821	0.268	1.5204
		0.259	1.5258

In both cases *A* and *B* the extinction was complete in the direction of the shorter wave-lengths as shown in Fig. 9, *b*; the extinction point is less definite in the plate for *B*. In cases *C* and *D* mixtures are encountered which come close to the condition deduced from equations (2) and (5), where the extinction travels very quickly through the whole spectrum. In both cases series of exposures were made with only half a minute angular interval, and the extinction angle is thus found which is valid within the accuracy of the experiment for all wave-lengths. A series of values of refractive index is then calculated for a number of arbitrary wave-lengths between 0.24 and 0.4 μ .

In the case of glycerine the extinction is complete on the long wave-length side of the extinction border of the spectrum, as in Fig. 9, *c*, which shows the result obtained in an experiment with castor oil. Curves for the whole series of glycerine-water mixtures are shown, reduced from a large scale graph, in Fig. 10. The change of refractive index with temperature in these mixtures is of the order of 0.00025 per degree centigrade.

One practical application of the above results is to determine the refractive index, for some line in the visible region, of a mixture (to serve as an immersion fluid) which has the same refractive index as fused quartz for the usual "ultra-violet microscopy" wave-length of 0.275 μ . For this purpose the refractive indices of the mixtures for 0.275 μ are plotted against the corresponding values for $\lambda = 0.546 \mu$ and $\lambda = 0.5893 \mu$ respectively. The resulting curves are very nearly straight lines; from them the following results are found for 20° C.:

$$N_{275} = 1.4961, \quad N_{546} = 1.4544, \quad N_{5893} = 1.4530.$$

Messrs R. and J. Beck, Ltd., have supplied figures which are understood to have been obtained with the Hilger ultra-violet refractometer apparatus; from these the corresponding value for 0.546 μ is obtained as 1.4541, which is in fairly good agreement. The above figure for 0.5893 μ will, however, enable the immersion fluid to be made up with the Abbe refractometer, using daylight. Moreover the curves will be useful if it should be desired to improve the optical system by the use of a liquid of any other refractive index; this is often a useful procedure in eliminating residuals of zonal aberration.

THE PHENOMENA OF THE EXTINCTION BAND

The phenomena of total reflection at a film of liquid pressed between the hypotenuse faces of right-angled prisms were early described by Newton; as is

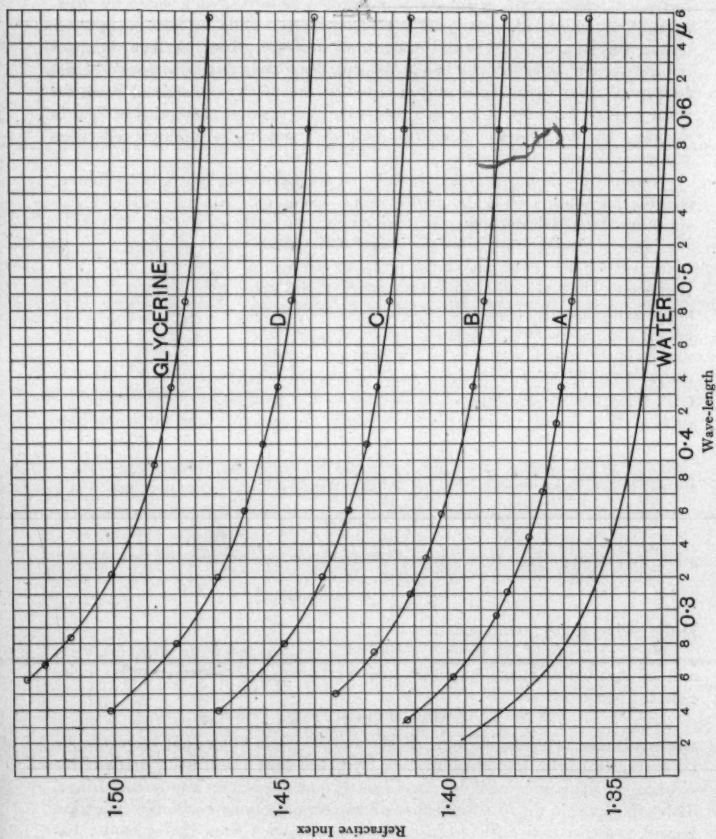


Fig. 10. Refractive indices of glycerine-water mixtures.

well known, the arrangement is used in such refractometers as that of Abbe. Christiansen, Mach, and others have employed such an arrangement for refractometry with visible light. If the image of the film be formed on a screen by transmitted light, and the illumination of any point observed while the angle of incidence on the film is made to increase by the rotation of the prisms, the light becomes coloured yellow, then red, and finally vanishes completely as a bow-shaped border sweeps over the screen. The objection to the use of the transmitted beam for refractometric purposes is, as Mach points out, the possible absorption of light in the film. For this reason he developed methods depending on multiple reflections at angles near the critical angle.

Nevertheless the transmission method has a considerable sphere of usefulness, and the phenomena near the border when using the apparatus described above may be of interest. We will consider the simplest case when the image of the border is formed by the refraction at the posterior surface of the sphere. The "monochromatic" boundary is of course not absolutely sharp; in the present case its location in the principal focal surface corresponding to the posterior surface of the sphere of quartz will be found by drawing the ray OB (Fig. 11) at the critical angle with the normal OO' . The point B describes a circular arc in the focal surface. If the radius of the surface is r that of the principal focal surface will be

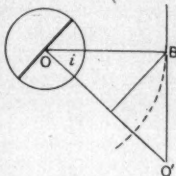


Fig. 11.

$$r + r/(N - 1) = rN/(N - 1);$$

the radius of the extinction border will therefore be

$$\frac{rN}{N - 1} \sin i,$$

while that of its projection on a screen touching the focal surface will be

$$\frac{rN}{N - 1} \frac{\sin i}{\cos i} \quad (\text{approximately})$$

in the region of the contact point.

If α is the angle of incidence of a ray inside the film and d the thickness, there will be destructive interference between the transmitted radiation and a twice reflected part when

$$2d \cos \alpha = (2n + 1) \lambda/2,$$

where n is 0 or a positive integer. At the critical angle $\cos \alpha = 0$; there can be no transmitted light, but when $2d \cos \alpha$ increases, with a decreasing angle of incidence, to $\lambda/2$, the growing light is again partly quenched by an interference fringe. This will appear in the focal surface as a dark fringe concentric with the extinction border, and consecutive fringes correspond to $3\lambda/2$, $5\lambda/2$, etc. Owing to the dispersion, these fringes are not visible with white light, but their effects are seen when the extinction band falls on the slit of the spectrograph.

A variety of interference phenomena, which are new to the writer, then appear

in the spectrum. They are a special case of the well-known fringes caused in the spectrum by placing a "thin film" in front of the slit.

In order to understand them, assume the border for 0.2313μ to fall on the slit. The angle of incidence in the quartz is then $38^\circ 30'$ when an air film is used. It is of interest to make a table showing the cosines of the angles of incidence in the air film for the longer wave-lengths which will all be transmitted. These are easily computed from a table of the refractive indices of quartz and are given in Table VII.

Table VII

$(\lambda) (\mu)$	$\cos \alpha \times 10^2$	$\cos \alpha \times 10^2 / (\lambda)$
0.2313	0	0 (approximately)
0.2446	4.69	19.17
0.2573	11.2	43.6
0.2749	15.3	55.8
0.3034	19.05	62.8
0.3303	21.2	64.3
0.3611	22.96	63.59
0.3962	24.31	61.36
0.4046	24.60	60.8

In the last column $\cos \alpha$ has been divided by λ , and it is on this that the "order" of the interference depends since

$$n = 2d \frac{\cos \alpha}{\lambda} - \frac{1}{2}.$$

The important feature of the table is the occurrence of a maximum in the last column; the occurrence of such a maximum near the extinction point is quite usual.

Suppose, for example, that we have the p th fringe for wave-length 0.3303μ falling on the slit; p is dependent on $(\cos \alpha)/\lambda$, which is then a maximum. For the p th fringe in wave-lengths on each side it is evidently necessary to seek a greater value of $\cos \alpha$, i.e. a diminished value of α . The effect is illustrated diagrammatically in Fig. 12.



Fig. 12.

In the right-hand portion are represented the p th fringes for wave-lengths λ_1 , λ_2 , and λ_3 , that for λ_2 lying at the position for greatest i , a maximum being assumed to occur here.

It is easily seen, by drawing the projection of the spectrum, that closed interference figures may be expected in the neighbourhood of the extinction border.

These are easily observed, and a photograph is shown in Fig. 13. Owing to the finite dimensions of the spectrum, only one or two of the closed figures can be observed. Between the centre of the closed curves and the extinction border are found a number of curved interference fringes. Naturally these fringes are best seen by using a source giving a continuous spectrum and observing with a very fine slit in the spectrograph or spectro-scope. The extinction border itself may be regarded as marking the centre of the first "bright" fringe.



Fig. 13.

NOTE ON METHOD (1)

In Method (1), where a very narrow first slit is employed, the angles of incidence of the radiation proceeding to various parts of the slit image are different. Let ON (Fig. 14) represent the central normal to the vertical film; OR is a ray in the horizontal plane travelling to the centre of the slit, while OP is a ray making an angle θ with the horizontal and passing through the point P in the slit. The angles $\angle NOR$, $\angle ROP$, and $\angle NOP$ are written i , θ , and ϕ . Then

$$\cos \phi = \cos i \cos \theta.$$

Fig. 15 represents diagrammatically the appearance when the extinction with such a liquid as water is photographed, the extreme extinction falling at λ_1 . The

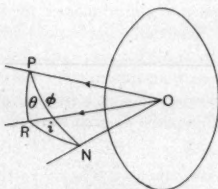


Fig. 14.

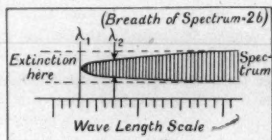


Fig. 15.

angle of incidence will be known from the circle reading, but this angle is valid only for that wave-length extinguished in the centre of the breadth of the spectrum. Other wave-lengths are extinguished at such a distance from the axis as suffices to bring the angle ϕ (Fig. 14) up to the critical angle. Hence if $2b$ is the breadth of the band at λ_2 we shall be able to calculate the angle of incidence (on the film) corresponding to the edge of the band. In the simplest case (quartz sphere only) it will be given by $\tan \theta = cb(N-1)/rN$, whence θ can be found. The constant c allows for the magnification. Then

$$N_I = N_o \sin \phi = N_o (1 - \cos^2 i \cos^2 \theta)^{\frac{1}{2}}.$$

In this way a single plate could be made to provide measurement only for λ_1 ,

the more or less accidental location of the extreme extinction (which cannot be controlled with any great certainty when photographs are taken in the ultra-violet), but also for any chosen spectrum line λ_2 in the neighbourhood of the extinction limit. Since this would involve additional linear measurements the question of the possible accuracy will not be discussed at present, but it will be clear that Method (1), using a specially built instrument, is probably capable of interesting developments. The above provides an alternative to the use of the spectroscopy slit at right angles to the axis of rotation of the prisms.

CONCLUSIONS

The drawback of the method used for the measurements described in this paper is the length of time necessary in taking a series of photographs with very narrow slits. Since the temperature must be controlled carefully if a reasonably definite extinction is to be obtained, the length of time, perhaps 20 minutes, in an experiment is rather long unless a thermostat is available. Otherwise there is no particular experimental difficulty. It seems not too much to claim that the corrected results obtained by the present apparatus are probably accurate to the third decimal place, and that when check results can be obtained by spectrometer measurements over the whole available range, the errors will be reduced to two or three units in the fourth decimal place.

If it were desired to improve the present arrangements as far as accuracy is concerned, a longer focal length for the objective would be tried. On the other hand, this would lengthen the exposures.

DISCUSSION

Dr R. S. Clay: With the extension of the use of the microscope into the ultra-violet, with the aid of quartz and fluorite lenses, it has become increasingly important to have a simple method of obtaining the optical constants of media for mounting and for use with immersion objectives in ultra-violet light, and the split sphere method devised by Prof. Martin enables these to be determined very simply and with sufficient accuracy for most purposes. Possibly the method which I demonstrated to the Society a few years ago—in which the film of the liquid, of which the refractive index is required, is formed between two small right angle isosceles prisms (see Clay's *Treatise on Light*, p. 136)—might equally be used for the purpose. This method would have the advantage over the method described by Prof. Martin in that the quartz prisms required could be more easily constructed than his hemispheres. The prisms should have their optic axes normal to the plane of incidence; axis cut quartz lenses would have to be substituted for the spectacle lenses described; and it would be advisable to look at the line of demarcation of the shadow through an analysing prism. The scale is a very open one.

Col. J. William Gifford (communicated): I have been much interested in Prof. Martin's well reasoned and intricate paper. But I would ask why the divided sphere is afterwards called a "prism" system. My idea is that it would be easier to follow throughout as a sphere. And I would suggest that the mixtures of glycerine and water would have been better indicated by their specific gravities than by weight percentages. I have worked a good deal with glycerine as a cementing medium for quartz-calcite and fluor-quartz lenses during the last 30 years and quite agree that it is very strongly hygroscopic. It has now been superseded for my own use by solutions of dextrose. This is not hygroscopic and, like balsam, does not easily dry; moreover, it may be prepared of the same consistency and may then be used as a cement. Thicknesses of 12 mm. and over are practically opaque to the ultra-violet but with those of the order of 0.025 mm. and under the absorption is negligible.

After all, the question arises, is all this complication necessary? In order to measure the refractive index of an immersion fluid for the ultra-violet with markedly greater accuracy, all that is necessary is an ordinary spectrometer with fluor-quartz lenses and a rather small hollow prism of quartz (or fluorite if available). Such a prism of glass with an opening of dimensions only 15×13 mm. is in my possession and holds one fluid drachm, less than which is scarcely likely to be made up of any immersion medium. When I add that the preliminary measurements of melted quartz made by the late Mr Shenstone and myself in 1904 were all made with a prism with faces only 10×13 mm., it will be seen that such an opening should be adequate. Measurements of the refractive indices of water for the ultra-violet have been made more recently than those quoted in the paper by myself. The apertures of the fluor-quartz lenses and quartz prisms employed were 37.5 mm. I shall be happy to send reprints containing the measurements to any members of the Optical Society who may be interested.

Prof. Martin: In reply to Dr Clay, the use of the double hemisphere system was intended in the first place to suffice for both prisms and lenses and, indeed, it is possible to obtain quite useful results by such simple means. When the requirements for an accurate ultra-violet method are considered, in which the border line *cannot be looked at* but must be found by photography (using a spectrograph) or (less certainly) with a fluorescent screen, it is very debatable whether a quartz prism system would prove any cheaper or more convenient than the one which has been described. Complete hemispheres were not required; it is certainly quite as difficult to produce a good plane surface as a true segment of a sphere; moreover, it would still be necessary to employ a fluor-quartz achromatic lens and a quartz collimator lens, so that no economy would be realised in the number of optical parts. On the other hand, the scale could be made somewhat more open. The numerical calculations would be more complicated with the prism device since tables such as can be used with the Pulfrich instrument could not well be employed. I think it would be difficult to say which arrangement is the better without practical experience of both; but to some extent the work described in the paper has elucidated the general requirements of any critical angle method.

In reply to Col. Gifford I shall certainly try solutions of dextrose in regard to their possibilities as immersion media, and this, I think, is a good example of a case in which a method like the one described in the paper (using only a thin film of liquid) will probably allow of a sufficiently accurate determination of the refractive index in the ultra-violet region desired, whereas the hollow prism method would probably break down at a much longer wave-length. The hollow prism is, of course, the only means when really great accuracy is required, but thin film methods certainly offer additional freedom in particular cases such as the above, and others which might be cited. A few comparatively inexpensive accessories to a small quartz spectrograph, plus an iron arc, will enable anybody to get a complete refractive index curve for the ultra-violet.

Perhaps the use of the word "prism" is not advisable, but I was anxious to avoid saying "sphere." The refractive indices of the mixtures in the visible are the most convenient specification. I think it is much easier to make up a mixture to a prescribed refractive index, measured immediately with an Abbe refractometer, rather than to a prescribed specific gravity.

I should like to thank both Dr Clay and Col. Gifford for their remarks, which certainly raise important questions to be borne in mind.

THE CONSTRUCTION OF WAVE-LENGTH SCALES FOR SPECTROGRAMS

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[Communicated by Dr John S. Anderson.]

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ABSTRACT. A method is described by which an approximate scale of wave-lengths may be projected geometrically from a uniformly divided scale on to a spectrogram whereon a sufficient number of lines have been identified to enable constants of a Hartmann interpolation formula to be derived. Such a scale is of value in assisting the recognition of other lines between which accurate interpolation may be required.

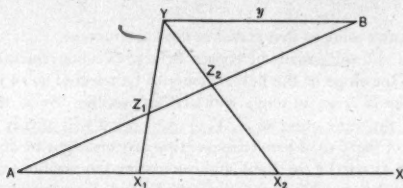
MANY types of spectrograph are not provided with any wave-length scale and spectrograms obtained with them require to be calibrated by identifying a number of easily recognisable lines, of which the exact wave-length may be obtained from published tables. Intermediate wave-lengths may then be interpolated from a graph or by means of Hartmann's formula $(\lambda - \lambda_0)(n_0 - n) = c$ when the constants λ_0 , n_0 , and c have been evaluated, provided the measurements of distance n are always made from a zero corresponding with the position of a standard line used in deriving the constant n_0 . Interpolation between two lines, λ_1 and λ_3 , whose distance apart is d , may, however, be more readily and more accurately performed by eliminating n_0 between the two corresponding equations and writing

$$\frac{\lambda_2 - \lambda_1}{\lambda_3 - \lambda_1} = \frac{x}{d} \cdot \frac{c - d(\lambda_1 - \lambda_0)}{c - x(\lambda_1 - \lambda_0)},$$

where λ_2 is the wave-length of the unknown line and x its distance from λ_1 .

The recognition of intermediate wave-lengths for purposes of calibration or of interpolation would obviously be facilitated if a scale of wave-lengths could be applied to the spectrogram, even though the scale were not of the highest accuracy. The writer has not come across any description of the method of construction of such a scale and offers the following in the hope that it may possibly be of use to others. It is assumed that the constants of the Hartmann formula have been derived with an accuracy sufficient to warrant its use over the range of wave-lengths in question.

Let AX carry a uniformly divided scale $x = k_1 (\lambda - \lambda_0)$ and let a unit k_2 be selected such that $k_2 c$ is a convenient length. From the known values of n_1 and n_2



corresponding with the positions of the lines λ_1 and λ_2 of the spectrum between which the scale is required, calculate the length

$$AB = \frac{k_2}{k_1} \left(n_0 - n_1 + \frac{k_1}{k_2} \right) \left(n_0 - n_2 + \frac{k_1}{k_2} \right)$$

and mark this length on a line making some convenient angle with AX . Through B draw BY parallel to AX and in the opposite sense: on BY lay out the length $y = k_2 c$. The projection of the scale of AX on the diagonal AB from the point Y is the scale $Z_1 Z_2$ required to cover the length of the spectrum between n_1 and n_2 .

The proof is as follows:

Obviously, for any index line YZX we have

$$\frac{y}{x} = \frac{BZ}{AZ}$$

Therefore

$$c = \frac{BZ}{AZ} \frac{k_1 (\lambda - \lambda_0)}{k_2}$$

Hence, if AB carry a scale such that

$$\frac{BZ}{AZ} \frac{k_1}{k_2} = n_0 - n, \text{ i.e., } AZ = \frac{AB}{\frac{k_2}{k_1} (n_0 - n) + 1},$$

the equation becomes the Hartmann formula

$$c = (n_0 - n) (\lambda - \lambda_0).$$

The length of AB is found by considering the two points Z_1 and Z_2 corresponding with the points X_1 and X_2 defined by the values of λ_1 and λ_2 . For

$$\begin{aligned} Z_1 Z_2 &= AZ_2 - AZ_1 = \frac{AB}{\frac{k_2}{k_1} (n_0 - n_2) + 1} - \frac{AB}{\frac{k_2}{k_1} (n_0 - n_1) + 1} \\ &= \frac{k_2}{k_1} AB \frac{n_2 - n_1}{\left\{ \frac{k_2}{k_1} (n_0 - n_2) + 1 \right\} \left\{ \frac{k_2}{k_1} (n_0 - n_1) + 1 \right\}} \end{aligned}$$

Now $Z_1 Z_2$ is required to be equal to $n_2 - n_1$. Therefore AB must be equal to

$$\frac{k_1}{k_2} \left\{ \frac{k_2}{k_1} (n_0 - n_2) + 1 \right\} \left\{ \frac{k_2}{k_1} (n_0 - n_1) + 1 \right\},$$

which has the same value as that stated in the construction.

The scale on AX will usually be chosen as large as is conveniently possible: the value of k_2 and the slope of the line AB should be selected so as to obtain index lines which make as large an angle with AB as possible. Since $AX_1 (= \lambda_1 - \lambda_0)$ will often be of the same order as $X_1 X_2 (= \lambda_2 - \lambda_1)$, it will usually be desirable to omit the points of the Z so as to save space: this may of course be done by selecting some angle such as $\tan^{-1} \frac{1}{2}$ for BAX and calculating the positions of the points in which perpendiculars to AX drawn through X_1 and X_2 intersect AB and of the point in which the latter perpendicular intersects BY : the last point in the case suggested is given by $BY' = \frac{1}{2} AB - AX_2$ and $X_2 Y' = \frac{1}{2} AB$.

DISCUSSION

Mr T. Smith: As an old member of this Society, and also an old colleague of Dr Barr, it gives me much pleasure to be able to welcome Dr Barr's first contribution to our *Transactions*. A good deal of optics enters into some of Dr Barr's work, and I hope we may receive other contributions from him.

The solution of the problem discussed in this paper appears to depend on the satisfactory character of the Hartmann dispersion formula. Ultimately, however, the problem is one of interpolation, and can be discussed independently of that formula. The basic assumption is that we have a one-one correspondence between two ranges of points situated on two straight lines. Algebraically this correspondence is expressed in the form

$$axy + bx + cy + d = 0,$$

and the relation is completely determined when we know three pairs of corresponding values of x and y . In its application to the present problem we may put $y = f(\lambda)$, and we see that the general Hartmann formula belongs to a group of expressions derived from the assumption of a one-one correspondence. The particular form $f(\lambda) = \lambda$ is adopted for its simplicity rather than its accurate representation of observations, and it is useful to know that Dr Barr has found this approximation satisfactory over a considerable range of the spectrum. The a of this equation is either zero or finite. In the former event x is a fixed multiple of y plus a constant, and the evaluation of x for a series of values of y is very simple, whether it be done numerically or graphically. When a is finite the relation is essentially the same as that between the conjugate foci of an optical instrument. A large number of graphical and mechanical constructions for finding these

conjugate points has been described. One of the simplest consists in bending the axis at the lens L as shown in Fig. A, and drawing through the two principal foci

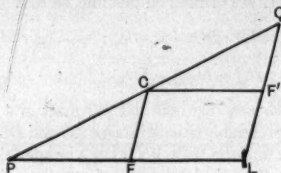


Fig. A

F and F' straight lines CF and CF' parallel to FL and $F'Q$; any straight line PCQ then meets the axes in conjugate points P and Q . For from the similar triangles

$$PF : FC = CF' : F'Q$$

or $PF \cdot F'Q = FL \cdot LF' = \text{constant}$, and this is Newton's optical formula. This only illustrates a slightly different arrangement of the construction Dr Barr is using. The essential fact is that graphically any one-one correspondence is equivalent to projection from a fixed point.

If I understand Dr Barr's problem aright, he wishes to read off wave-lengths on a print of a spectral photograph after he has identified the wave-lengths of three lines. If measurements have in any event to be made, the calculations may be made very quickly to any required accuracy, using a table of reciprocals. Nothing more than adding a constant to a reciprocal is necessarily involved. When a graphical

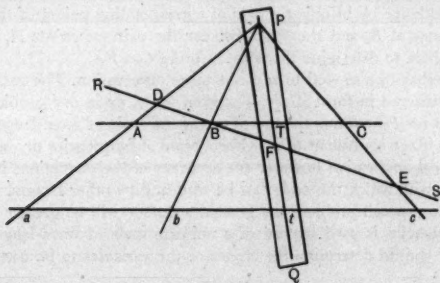


Fig. B

method is preferred, and particularly when many photographs have to be examined, the following routine, which involves no calculations, offers some advantages.

The photograph is fixed on a drawing board either over a sheet of plain paper or under tracing paper, in a suitably inclined position. The spectral lines are read where they meet a line RS (Fig. B) drawn on the photograph. The points of inter-

section of RS with spectral lines of wave-lengths $\lambda_1, \lambda_2, \lambda_3$ are D, B, E respectively. Through B a straight line is drawn perpendicular to the guiding edge of the drawing board, and points A and C are marked on this line, so that

$$AB : BC = \lambda_1 - \lambda_2 : \lambda_2 - \lambda_3,$$

or more generally $AB : BC = f(\lambda_1) - f(\lambda_2) : f(\lambda_2) - f(\lambda_3)$.

The distance AC is so chosen that AD and EC meet at a point P conveniently placed on the board and preferably situated above AC and DE . The divided scale ac of wave-lengths, either a uniform scale if $f(\lambda) = \lambda$ or specially graduated if this is needed, is now placed on the board perpendicular to the guiding edge (and therefore parallel to AC) with the graduations a, b, c corresponding to the wave-lengths $\lambda_1, \lambda_2, \lambda_3$ lying on PA, PB , and PC respectively. It is now only necessary to take a transparent glass or celluloid rule on which a straight line PQ passing through a small hole P has been engraved, and ensure that this hole coincides with P , the intersection of AD and EC , by passing a drawing pin through the hole and inserting it in the board at P . The transparent rule can now be moved about P until its engraved line passes through F , the intersection with RS of the spectral line it is desired to identify, and the wave-length is then read on the scale by the point of intersection t of the engraved line with ac .

It will sometimes be convenient to let AC and the scale coincide, the wave-length for the line F then being read at T , but the routine described is more general.

The accuracy of the method follows from the facts that the desired conversion is attainable by rectilinear projection, and that the correct projection is fully determined by securing correspondence for three distinct pairs of points. We obtain one correspondence by bringing a pair of corresponding points of the two scales into coincidence at B , and the other two for the pairs of points A, D and C, E by utilising them to determine the point of projection P .

It may perhaps be as well to add one more observation. The general one-one relation is unaltered in form if $1/y$ is written for y , or in our problem if $1/f(\lambda)$ is substituted for $f(\lambda)$. Thus the use of a uniform scale of wave-lengths is mathematically the exact equivalent of a uniform scale of frequencies or wave numbers. In the practical application however the accuracy of the deductions is affected by the closeness with which the scale can be read and by other factors. It is easy to see that in this special problem the general accuracy will be greater if a uniform scale of frequencies is used instead of a uniform scale of wave-lengths, and this consideration should determine the choice of the variables to be used.

Dr R. S. Clay: Dr Barr's paper is to be welcomed as it explains to the user of the spectroscope an easy method of calibrating the instrument, which should be sufficiently accurate for all ordinary purposes. As many to whom the method would be of the greatest use are practical men who would be interested not so much in the theory as with its application, I hope that Dr Barr and Mr Smith will add a paragraph explaining in the simplest and most straightforward way exactly

how the method is to be applied in an actual instance. If a numerical example could be given, it would probably be an advantage.

Dr Barr: I am grateful to Mr Smith for the general treatment which he has explained for the problem. I had a suspicion that there should be a graphical method of construction which would avoid the necessity for calculating the constants of the Hartmann formula. I suggest, however, that the method which I have indicated is probably the more accurate in practice, if it is to be assumed that $f(\lambda) = \lambda$, since it locates the point P , from which the projections are to be made, by calculation rather than by finding the point of intersection of AD and EC of Fig. B.

ON THE RESOLUTION OF GRATINGS BY THE ASTIGMATIC EYE

By J. R. HAMBLIN, B.Sc., AND T. H. WINSER

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ABSTRACT. An investigation on the resolution of gratings by the human eye, commenced by H. H. Emsley in 1925, was continued. The irregular curve representing variation of grating acuity from meridian to meridian was found to be of the same general form in all astigmatic eyes. Experiment showed that the image, produced by an ordinary astigmatic system, of gratings placed in various directions shows irregularities of the same form due to overlapping of the blurred images of the separate lines of the grating. Thus an eye with regular astigmatism will exhibit irregularities in its capacity to resolve gratings at various inclinations, and charts consisting of fine parallel lines are not reliable in testing ocular astigmatism.

1. THE DETERMINATION OF GRATING ACUITY FOR VARIOUS MERIDIANS

In a paper* read before the Optical Society in November, 1925, Mr H. H. Emsley described a method he had employed for studying the acuity of the eye in different meridians. The variation of the acuity from meridian to meridian in both unaided and corrected eyes suggested that further consideration of the problem was desirable. Accordingly Mr Emsley asked us, as students of the Northampton Institute, to conduct more experiments with the same apparatus. The following is a record of the experiments carried out.

The apparatus employed for the determination of visual acuity was identical with that described in the paper mentioned above. It consists of one or more circular plane glass discs, ruled with a black and white grating, made photographically, and illuminated from behind through discs of frosted glass and further discs of opal glass. They are capable of rotation through 180° from behind and are mounted in a blackened wooden box, so that no extraneous light reaches the observer. In the apparatus as constructed three discs were employed, mounted at the corners of an equilateral triangle, but one of these was a dummy and was never used. The apparatus was suggested by Mr E. F. Fincham for the determination of the alteration in acuity with ametropia.

The methods of use of the apparatus were as follow. In the first method the observer whose eye was being tested maintained a fixed position, and drew towards him the box holding the gratings, which was mounted on a suitable carriage. As soon as the lines were just recognised, their direction was indicated to the other

* H. H. Emsley, "Irregular Astigmatism of the Eye: Effect of Correcting Lenses," *Trans. Opt. Soc.* 27, 1925-6 (28).

observer. If the direction indicated was correct, it was recorded, together with the distance at which the lines were first recognised. The direction actually noted was the meridian of the eye employed in resolving the grating, i.e. perpendicular to the direction of the lines. In the second method the apparatus was kept in a fixed position while the observer moved up towards it.

It was found in practice that the second method was more convenient, and indeed quicker, since the observer could move more surely than he could control the movement of the box, and the first method was impracticable with the apparatus set up, where there was a large difference in the distances at which the grating was recognised. Since the observer had to indicate in what direction the lines were, he could presumably resolve the grating and, as the observer setting the grating varied its direction indiscriminately, a fair test of acuity should result.

Preliminary tests on the authors' eyes were carried out in order to become conversant with the apparatus and its use. It was found necessary to obtain for

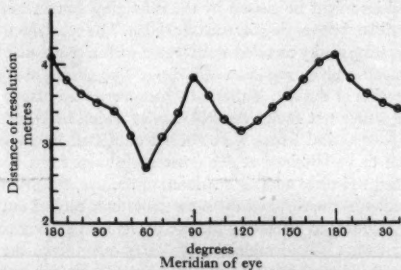


Fig. 1. J. R. H. Left eye: Myopic Astigmatism

each meridian (taken at intervals of 10°) a set of readings of the distances at which the grating was resolved; also no reliance could be put on a single reading, three being necessary in most cases. A single reading in a set obtained for a given meridian of an eye may differ by as much as 20 per cent. from another reading in the set, all others obtained lying between these extremes. In general, this occurred when the distances at which resolution of the grating was obtained were small, i.e. in the neighbourhood of 3 metres and also for certain meridians in some eyes, where the observer had difficulty in deciding whether he could see the grating or not. It was also found that average numerical values of the distances of resolution for a given meridian and observer, while preserving approximately the same relative values, varied slightly on different occasions. This variation amounted to as much as 5-7 per cent. Consequently a complete set of readings had to be obtained at one "sitting" and as rapidly as possible to obviate fatigue effects. To demonstrate the varying acuity, the distances found are plotted graphically against the meridians of the eye employed.

Fig. 1 shows a curve obtained by this method. The curve for a perfect eye would

be a straight line with constant ordinate, whereas this curve has practically no consecutive points having the same value of ordinate. Its chief characteristic appears to be the fact that there are two maximum acuities at right angles to each other, and of almost equal value, separated by two minima of differing degrees of "badness." Similar curves were obtained for the other eyes tested with unaided vision. No case demonstrated a perfect eye or even a regularly astigmatic eye, since we considered that this should give a regular curve.

2. THE EFFECT OF CORRECTING LENSES ON GRATING ACUITY

Since it appeared that these cases proved the existence of irregular astigmatism, it was thought that something might be learned if, instead of attempting the subjective correction of the whole pupil by spherical and cylindrical lenses, correction should be made on the pupil, meridian by meridian, on the supposition that the irregularities might be caused by the refracting system having a constant value across a meridian but varying for each meridian. The eyes which had previously been tested for grating acuity unaided were tested with a stenopæic slit in position before them to isolate a given meridian of the eye. The grating was placed at right angles to the direction of the slit. Spherical lenses were then placed before the slit and the observer under test found the distance at which he could just recognise the grating. These spherical lenses were altered until that lens was found which enabled resolution to be obtained at the greatest distance from the grating. The slit was then turned to isolate another meridian of the eye, the grating again being placed at right angles to the slit, and the same procedure carried out for every 10° . This experiment amounts to refracting the eye for its best grating acuity 18 times, and consequently took a considerable time to carry out. Since the grating stood at right angles to the direction of the slit, the observer under test knew in what direction the lines should appear, and it was thought that this knowledge might invalidate the results. However, with care, consistent results were obtained and, on repetition some weeks later, the results were almost exactly equal. Only two eyes were tested in this manner, as so much depended on carefulness of observation and the integrity of the observer.

The results obtained by this method, in the examination of the same eye as gave the results in Fig. 1, are shown in Fig. 2. It will be seen that resolution of the grating is obtained at much greater distances, the maximum being 9 metres, and is equal in the 90° and 180° meridians where it is best, but in the meridians between, it decreases irregularly from the maxima to minima at 55° and 130° , agreeing with those minima found for the same eye uncorrected, viz. at 60° and 120° (Fig. 1). A noticeable difference between the curves is the alteration in relative importance of the minima. In Fig. 1 the 60° meridian has by far the worst resolution; in Fig. 2 this occurs in the 130° meridian. In Fig. 2 is also plotted the curve of the powers of the lenses employed before the slit to obtain the maximum distances of resolution referred to above. This curve of lens power has a single maximum at 90° and a minimum at 170° . As comparison with this curve, a curve

showing the variation in power of a sphero-cylindrical lens, whose maximum and minimum powers (i.e. on its axes) coincide with those of the curve of lenses before the slit at 90° and 180° , is also plotted in Fig. 2. The curve of lens powers obtained can only be considered as a rough approximation to that of the sphero-cylindrical lens, since two straight lines, one drawn from the 170° value (2.62 D.) to the 90° value (4.62 D.) and the other from the 90° value to the 170° value in the opposite direction, form a closer approximation to the curve than that of the sphero-cylindrical lens. In the case of the latter, the values of the lens powers at 80° and 100° are only very slightly less than that at 90° , while in the experimental values obtained on the eye a considerable difference is seen. Again, while the two curves coincide at 90° and 180° , the power of the sphero-cylindrical lens exceeds that

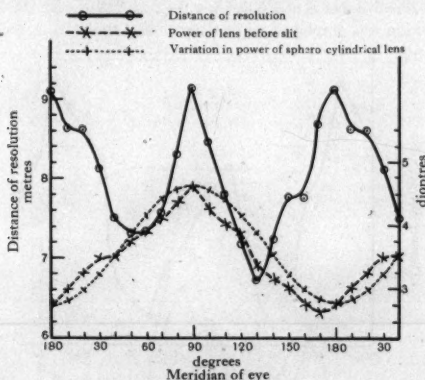


Fig. 2. Same eye as in Fig. 1

of the experimental curve from 50° to 90° and from 90° to 180° , being slightly less from 180° to 50° . The cylindrical power of this sphero-cylindrical lens being 4.62 D. - 2.75 D. = 1.87 D., in the subjective correction of the eye it would be expected that a cylinder of less power than this would be accepted unless the eye would tolerate the over-correction resulting in several meridians.

In the other eye studied by this method of correcting the meridians of the eye separately, the astigmatism found was quite small (0.75 D.) and the accuracy of the method was not sufficient to demonstrate the deviations (if any) from the true cylindrical correction.

Since the eye whose behaviour has been followed in detail seemed suitable for further investigation, a very careful retinoscopic examination was carried out. Allowing for the working distance (1 metre), this gave -3.00 D.SPH. with -1.5 D.CYL. or -2.0 D.CYL. There is a large amount of uncertainty in this knife-edge test owing to the aberrations. With dynamic retinoscopy, allowing for

0.50 D.SPH. lag, the result was -3.00 D.SPH. with -1.25 D.CYL. A very careful subjective test was then carried out. Using a single line astigmatic fan with between -2 D.SPH. and -2.5 D.SPH. before the eye, the vertical lines are the blackest. On adding correcting cylinders, in gradually increasing power, the fan becomes generally clearer and the lines on either side of the vertical get blacker. With -1.25 D.CYL. in position the vertical line and lines 20° on either side of it are fairly black. With -1.50 D.CYL. the vertical is slightly grey and lines at 60° and 120° are blackest, and with -1.75 D.CYL. lines at 40° and 140° approximately are blackest. Thus -1.50 D.CYL. is too strong since it over-corrects the eye for the vertical line, and even with -1.37 D.CYL. there is a tendency for this to occur. The subjective correction for this eye was found to be -2.75 D.SPH. $\ominus -1.25$ D.CYL. 175° or 170° . The eye was actually wearing -2.25 D.SPH. with the latter cylinder and this correction was employed while determining the grating acuity.

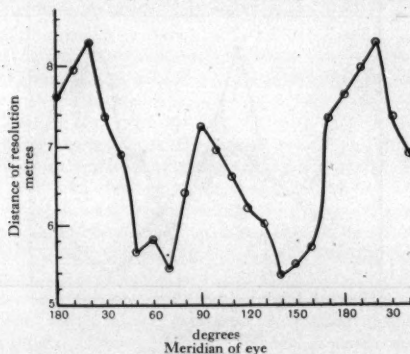


Fig. 3. Same eye as in Fig. 1, corrected by -2.25 D. Sph $\ominus -1.25$ D. Cyl. Axis 170°

Fig. 3 gives the results obtained as described above. The general form of the curve has not changed. The maximum is at 20° , a smaller one being at 90° , while the minima are approximately equal at 70° and 140° . It will be seen that the cylinder does not correct at the 90° meridian fully, since the grating acuity is less than that at the 20° meridian, so that a slightly better correction was obtained with the stenopaic slit method.

3. THE EFFECT OF A PINHOLE APERTURE ON GRATING ACUITY

Since the methods detailed above had failed to obtain equal resolution in all meridians, it was thought that some of the error might be zonal, and that a pinhole of small diameter, while removing the error due to astigmatism as determined previously, would also remove this. The two eyes above mentioned were tested for resolution of the grating with a pinhole (1.5 mm. diameter), placed centrally

with regard to the visual axis, as determined by the observer under test. Fig. 4 shows the grating resolution curve obtained for the same eye as gave the curves in Figs. 1, 2, and 3. From these curves it is to be noticed that, while the maximum distance is practically the same with the naked eye (Fig. 1) and with the pinhole (Fig. 4), the minimum distance is considerably greater in Fig. 4, and that the pinhole causes a general evening up of the distances. The general nature of the curve remains, the minima at 60° and 120° and the flattenings in the curves at 30° and

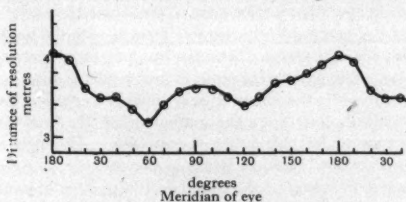


Fig. 4. Same eye as in Fig. 1, with pinhole

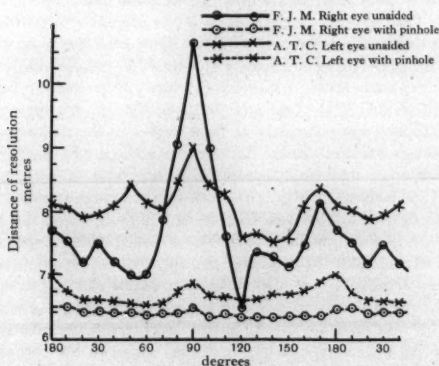


Fig. 5. Meridian of eye

140° being noticeable. This indicates that even a small area (1.5 mm. diameter) of the pupil has not only its regular astigmatism but also a certain amount of irregular astigmatism, a fact which Donders discovered many years ago by observation of small bright sources of light through a pinhole.

In view of the interesting nature of these curves, it was decided to test the resolution of a number of other eyes, to ascertain, if possible, the generality, or otherwise, of the results obtained. In each case the unaided eye resolution was obtained and also the pinhole resolution. Fig. 5 shows the results obtained on two

subjects, both having slight hypermetropic astigmatism. In both cases a remarkable increase in distance of resolution is displayed at 90° and a smaller increase about 180° , coupled with a sudden decrease to the lowest minima of the meridians at about 60° and 120° . With the pinhole (1.5 mm. diameter) in position, resolution is much more uniform in all meridians. In one case investigated (F. J. M. in Fig. 5) almost perfectly uniform resolution in all meridians resulted, the maximum difference being 16 cm. between the resolution in any two meridians. This difference, corresponding to not more than 2.5 per cent. of the actual distance of approximately 650 cm., is in marked contrast to the results obtained with the unaided eye where there was a difference of 4 metres (between 6.50 and 10.50 metres). Each point on the curve F. J. M. with pinhole is a mean of four readings differing in most cases by a few centimetres only, the observer being unable to resolve the grating until he reached a certain distance from the grating, when the lines flashed out and enabled the distance to be obtained with precision. Although a considerable number of other observers' eyes was tested in this manner, in no other case was such a near approach to a perfect eye found with a pinhole in position, although all observers obtained more consistent results with the pinhole in position; with some observers in particular, values obtained with the naked eye without pinhole varied widely. The considerable diminution in the average acuity with the pinhole is presumably due to the retina receiving less light, with the result that it is less sensitive to small differences of intensity and thus less able to resolve the lines.

The similarity immediately noticeable in most curves was the rapid decrease in acuity on either side of the best meridian, and the presence in every case of a subsidiary maximum, approximately at right angles to the major maximum. In addition, it was seen that the "kinks" in the curves could not be due to experimental error altogether, since they were reproduced to a greater or less extent in every eye examined, and consequently there must be something fundamental about them. With regard to the curves obtained when using a pinhole before the eye, these all demonstrated the fact that the large variations obtained with the naked eye, which were reduced considerably by reducing the aperture of the pupil, were due to refractive errors or possibly irregular diffraction effects and not to any geometric structure of the retina or interpretation of the retinal image by the brain.

Observations were taken on one eye having considerable irregular grating acuity to ascertain if the variations in acuity had their origin in the cornea. Owing to the fact that most other keratometers are designed only for ascertaining the curvatures of the principal meridians of the astigmatic cornea, the Ettles-Curtis instrument was employed. Allowing for experimental error, an almost truly smooth curve was obtained for the variation in curvature of the cornea in different meridians, showing that, in this case at least, the irregularities in grating acuity were not due to the corneal astigmatism departing from a regular form.

4. GRATING RESOLUTION BY AN OPTICAL SYSTEM WITH LOW REGULAR ASTIGMATISM

At this stage the close similarity of the acuity curves for all the eyes tested, as well as those which were studied by Mr Emsley, caused us some doubt as to whether true irregular astigmatism was being recorded. That is to say, although the pinhole test appeared to show that the irregularity of the acuity for different meridians was due to optical defects, it seemed improbable that these irregular defects would be of the same nature in all eyes. It is of course possible that similar irregularities may reside in such a structure as the crystalline lens in all eyes, but we have been unable to find a relation between the known anatomical structure and the experimental results obtained.

It was suggested that a better understanding of the significance of the curves could be obtained by comparing them with the performance under similar conditions of a model eye of known regular astigmatism. Such a comparison would test the reliability of our method of determining the effects of irregular astigmatism and would also help us to isolate the irregularities from the effects of regular astigmatism.



Fig. 6

Accordingly an apparatus forming a model eye was used as follows. An optical lantern *A* (Fig. 6), illuminated by a "Pointolite" lamp *B* (an arc-lamp was employed on one occasion), projected a beam of light through a slide *C*, consisting of a black and clear photographically-made grating capable of rotation in all meridians by a handle at the side. The image of the grating *C* was carefully focussed on a ground-glass screen *D* at a short variable distance away. At a farther distance, usually in the neighbourhood of 3 metres, was placed a camera *F* (a Thornton Pickard camera fitted with a Cooke $f/4.5$ lens). The image of the grating on the ground-glass screen *D* was focussed on the screen *G* of the camera by the lenses *H* and observed (because of the smallness of the image) by a low-power microscope *J*, through which the lines appeared comparatively large. A very weak cylindrical lens *L*, as low as 0.12 D., was placed over the camera lens, and the image of the grating observed through the microscope, as the grating was moved to different axes with respect to the cylindrical lens. If the cylinder *L* were initially placed at right angles to the grating lines and the camera focussed for best imaging, on slowly rotating the grating, the lines gradually became more and more blurred, and consequently less visible, until if the distances of camera from ground-glass screen *D*, size of grating on screen *D*, and (if necessary) power of cylindrical lens *L* were so adjusted, the lines in the image completely disappeared before the grating had been rotated

through 90° . Further rotation of the grating brought the lines gradually into view again to reach a maximum of distinctness and again disappear and even reappear again when the full 90° from the initial position had been accomplished.

Since the grating image on G when observed through the microscope is quite large, the question of the resolution of fine structure by the eye does not arise. The disappearance is caused by the intensity of illumination being actually sensibly uniform instead of varying between maximum and minimum values for the white and black lines of the grating. In other words, the resolution or non-resolution of the grating depends on the lens system $L + H$ and not on the observing eye, except in so far as the light sense enters.

The experiment described above does not reproduce the original experiments exactly, since in the latter the distance of the eye from the grating varied for different meridians, while the image remained of the same distinctness (for in each observation it was just resolved), but in the former the camera remained stationary and the image varied in distinctness.

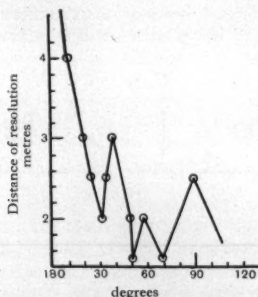


Fig. 7. Angular rotation of grating

An attempt was made, however, to reproduce in the following way the conditions under which the human eyes had been examined. The distance between the screen D and the lantern A was increased so that the image of the grating on D was so large that the image on the screen G of the camera was large enough for easy visual observation when the camera was 4 metres from D . For the initial position the axis of the cylinder L was placed vertically and the lines of the grating made horizontal and focussed accurately on G . When L was -0.50 D.CYL. on rotation of the grating through 10° , the lines of the grating observed at G disappeared. The camera therefore just resolved the grating when standing at 10° at a distance of 4 metres. The camera was then moved to a distance of 3 metres from D , the lines of the grating made horizontal, the camera refocussed, and the grating turned as before. In this case the grating at G disappeared at 21° and reappeared at 37° . This was carried out for three other distances and the results are given in Fig. 7. The results are very approximate, as it is extremely difficult

to decide on the exact position where the lines of the grating merge, but they are sufficiently accurate to show the character of the curve. It will be seen to represent a high acuity for one meridian (0°), a rapid decrease to a minimum at 30° , with two subsidiary peaks. This is sufficiently like the curves obtained for the human eye to suggest a similar origin.

With a fixed distance from the screen *D* to the camera, the camera images with the grating in various orientations were photographed, the illustrations (Fig. 8)

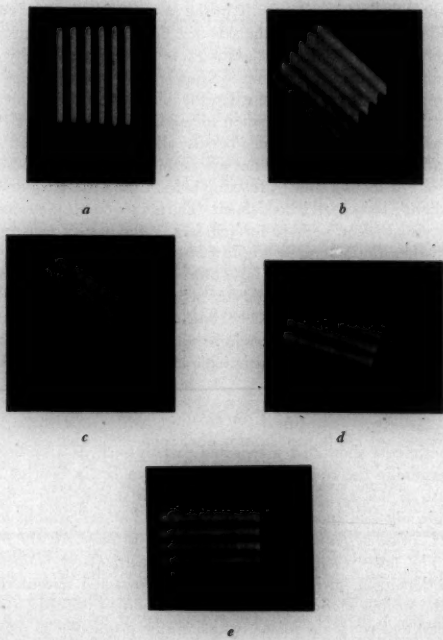


Fig. 8

having been prepared from prints made by Mr Fincham. These show clearly the varying distinctness as the grating is rotated. In the photographs the major portion is part of the large grating employed, and the faint small grating is the image of an extremely narrow strip of the grating isolated by pasting a strip of opaque paper over the original.

Referring to the large grating, it will be noted that, whereas in the original there are six white lines and until the first disappearance (Fig. 8, c) six are still recognisable although blurred, after this occurs, only five white lines are visible, showing that a spurious resolution has been obtained.

The small grating appeared on the original ground glass *D* as a row of illuminated slits of equal length separated by similar dark spaces, but, owing to the action of the cylinder *L*, appeared on the camera screen *G* as a short faintly illuminated grating. The photographs show how, on rotation of the grating, the slits are extended in a constant direction (vertically in the case considered), and, as the angle between the lines of the grating and the axis of *L* decreases, get closer together and would, if longer, overlap. In the large grating this overlapping actually occurs and, when it is of sufficient amount, the lines disappear. The actual amount of overlap to produce disappearance seems to be when the blurred width of a line is between two and three times that of the line when focussed, i.e. when vertical. Observers do not agree as to when disappearance occurs, and it is complicated by the fact that, if the grating is turned till no lines are visible on the camera screen and the eye is thrown out of focus a trifle by relaxing or increasing the accommodation, the lines become slightly but appreciably defined. The photographic plate does not quite agree with the visual effect, perhaps through lack of the perfect register required.

Obviously, when the blurred width of a bright line is so great that it overlaps considerably the adjacent bright line (also blurred), the sum of the two intensities will give the effect of a bright line midway between the lines considered. This is shown in Fig. 8, *d* and *e*, and explains the loss of one bright line of the grating.

From the above experiments we concluded that irregularity in the grating acuity occurs in the imaging of a regularly astigmatic lens system.

5. CONCLUSION

These experiments show that a repeated pattern such as a grating is unsafe to use in determining the resolving power of the eye, or, in fact, of any optical system, because of this spurious effect of resolution shown in the photographs. This effect is already well known to microscopists and others, who have tested the performance of optical systems in resolving repeated patterns, e.g. the markings on diatomaceae. The false resolution effect becomes particularly noticeable in the presence of astigmatism, giving the appearance of marked irregularities in the resolution of the various meridians, whereas the resolution varies in a regular way between the best and worst meridians, which are perpendicular to one another.

The practical bearing of these results lies in their application to the design of test charts, since a number of these have astigmatic fans, consisting of two or more parallel black lines on a white ground, at every 10° , and diamond or block charts, consisting of a number of black and white lines of equal width, forming a grating. It is necessary to consider briefly the methods of using these charts.

In ordinary subjective testing, the eye is rendered myopic, if not already so, so that (presuming astigmatism) both focal lines lie in the vitreous, one being quite

close to the retina. If the astigmatic fan be now observed, those lines which lie parallel to the focal line nearest the retina will be clearly defined, while other lines at other angles to this line should be correspondingly blurred. Reference to Fig. 1, which shows a curve obtained under these conditions, demonstrates the comparatively surprising result that grating acuity in the two best meridians at right angles is almost equal, although the cylindrical correction required (see Fig. 2) is 2 D. Thus the relative distinctness between the two principal meridians is not large, as is required for routine subjective testing. If a simple fan is employed consisting of single black lines every 10° , the lines are just blurred and no overlap can occur, provided that the lines are nowhere closer than 3 or 4 times their width. The astigmatic effects are then more clearly shown, an important consideration when testing unobservant patients. There is a tendency if lines appear black, say at right angles to each other, to call the whole fan black. This effect seems to be avoided with the single-radiating-line type of fan and the latter should be employed in preference to any other type where grating acuity has any influence in its imaging.

With regard to the diamond or block test, provided the approximate axis is known, this can be very useful, since the grating acuity diminishes very rapidly to minima within $30-40^\circ$ of the best grating acuity in practically all cases. In practice, then, it should not be swung more than this on either side of the approximate axis, and if the block consists of a grating formed by two sets of parallel lines at right angles, attention should be directed to one set only. With these precautions, the block test seems a useful and delicate one.

In conclusion, we must thank Mr Emsley and Mr Fincham for suggesting the work, and for guidance and helpful interest at all stages, as without their efforts this paper could not have been written.

DISCUSSION

Mr E. F. Fincham: The need for a suitable test object for the measurement of the resolving power of the eye has been keenly felt by all who have attempted to determine the effect of refractive errors upon visual acuity. By carrying out repeated tests with a simple grating of equal spacing, the authors of this paper have shown that this type of object is unsuitable as a test for acuity.

The phenomenon of false resolution, due to the overlapping of the blurred lines to which they have called attention, is well known to the microscopist but I think has not been mentioned before in connection with visual tests.

Some of the tests for visual acuity which are at present in use, particularly the Ives test, in which the subject has only to recognise the direction of lines in a grating, would appear to be unreliable in the light of the results given in this paper.

Apparently this effect of false resolution is liable to occur with many types of simple repeating patterns and, for this reason, such designs should be avoided as resolution tests.

Mr T. Smith: The authors of this paper have carried out a very interesting piece of work which will repay careful study. The selection of a satisfactory test

for acuity of vision in the presence of astigmatism is not as simple as it may at first sight appear, and I think objections, more or less serious, could be urged against almost any test that might be proposed. The remarkable thing about the observations recorded here is that they should have been so consistent, rather than that the distances of resolution of the corrected eye should be so variable. In a test of this kind it is to be expected that the observations will yield results expressible in the form

$$a + b \cos 2(\theta + \beta) + c \cos 4(\theta + \gamma)$$

together with small terms involving larger multiples of θ . The presence of an appreciable term in 4θ in experiments of this kind, where resolution is the criterion adopted, is not necessarily an indication of irregular astigmatism: for the optical path involves terms in 2θ , and the image intensity therefore terms in both 2θ and in 4θ , and resolution depends essentially on variations of image brightness.

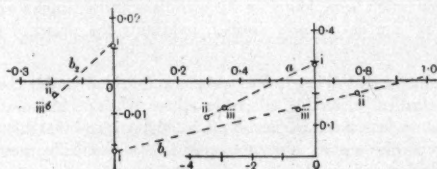
The authors have plotted their observations in a way which shows very clearly the variability of the distance at which resolution takes place, but the real significance of the results is somewhat difficult to grasp from such diagrams. Our attention is so much engrossed by the irregularities that we hardly appreciate the regularities. Instead of taking as a variable the distance at which resolution is obtained, we may consider diagrams with the reciprocal of this distance as ordinate. This gives us on one scale the curvature of the wave entering the eye, and on another a very fair measure of the distance apart in the ocular image of the lines which are just resolved. Instead of plotting these new diagrams we may analyse the results and express them in the form given above. Taking the diopetre as the unit, and calling the experimental results exhibited in Figs. 1, 2, and 3 of the paper cases i, ii, and iii respectively, I find the following values:

	Case i	Case ii	Case iii
a ...	0.290 (0)	0.127 (-3.42)	0.155 (-2.875)
b ...	-0.025 (0)	-0.005 (0.80)	-0.013 (0.625)
β ...	13° (0)	22° (7°)	21° (10°)
c ...	-0.026 (0)	-0.014 (-0.14)	-0.025 (0)
γ ...	4° (0)	2° (-13°)	12° (0)
b_1 ...	-0.022 (0)	-0.004 (0.78)	-0.009 (0.59)
b_2 ...	-0.011 (0)	-0.004 (-0.19)	-0.008 (-0.21)
c_1 ...	-0.025 (0)	-0.014 (-0.09)	-0.017 (0)
c_2 ...	-0.007 (0)	-0.002 (0.11)	-0.018 (0)

where b_1, b_2, c_1, c_2 are the coefficients of $\cos 2\theta, \sin 2\theta, \cos 4\theta, \sin 4\theta$ in the expanded form of the resolution expressions, and the quantities in brackets are the corresponding figures for the correction worn in the experiments. The particulars of the corrections in cases i and iii are given in the paper almost in the form quoted. For case ii the individual results plotted in Fig. 2 have been analysed. The outstanding discrepancies are small, except at 30° and 150° where they amount to $\frac{1}{8}$ D., 110° with the error $-\frac{1}{8}$ D., and 90° with an error of $\frac{1}{8}$ D.

There is not much that can be usefully said about the terms which involve 4θ . They account for the greater part of the distance variation, but from the small values of the amplitudes we can hardly attach great importance to them. There

is no clear connection between the values of these coefficients obtained in the three experiments. They vary, as the eye configuration changes, in some way which would only be revealed by a much more extensive series of observations. The case is quite different as regards the other coefficients. The accompanying figure shows for the variables a , b_1 , and b_2 the relation between the component of the correction and the corresponding component of the dioptric resolution curve (ordinates). They suggest that the real connection in each case may be linear. On this supposition we can write down the specification of a cylindrical correction which would yield a resolution curve entirely free from a component in 2θ , viz. $b_1 = 0.95$, $b_2 = -0.13$, that is $0.96 \cos 2(\theta + 4^\circ)$. This would correspond to a -2 D. cylinder with axis 175° . The sphere to be worn with it would be -2.5 D. or slightly less. It may be noted that the nearest single correction to case ii is in the usual notation -2.625 D.SPH. $\ominus -1.625$ D.CYL. axis 175° .



From the analysis of the observations it is clear that the planes containing the focal lines rotate as the distance of the object varies. In other words, the crystalline lens of this eye is evidently astigmatic.

It would be of value to have further series of these observations taken with this eye wearing different corrections. I very much hope that these further observations can be made. The three series I have discussed suggest that the connections between acuity of vision, the corrections worn, and the way in which the eye adapts itself to variations in the corrections, are simple. Three is the minimum number of series from which we can learn anything. From six or more suitably spaced series we might learn much more. In the absence of such observations suggestions concerning the relations between the a and b curves are mere speculation.

Comdr. N. M. S. Langlands: This paper is interesting. The problem is, however, complex. If sections through the volume diffraction image be considered, very different distributions of light are obtained from those given by sections of Sturm's conoid. In 1923 the speaker made at the Imperial College a series of photographs of the astigmatic diffraction images of a point of light. For small phase differences (0.25λ), using monochromatic light, the astigmatic geometrical line images were surrounded by elliptical rings. For phase differences of 0.75λ the ellipses become mainly concentrated in two bright areas on each side of each line, and for phase differences of λ these concentrations fade away and the line broadens and develops a hole at its centre. The intermediate sections are also very complicated.

To resolve any object, the eye will focus that section of the image which gives the best resolution. The resolving power for two points of light seen through such a system is intractable theoretically: for a grating the difficulty is still greater.

Furthermore, as the unaided eye approaches the grating, the pupil diameter will decrease.

There is no evidence of any marked variation of the resolving power of the eye in different meridians. This is also true of the resolving power for a vernier object. The effect of a rotated astigmatic lens on the vernier acuity has been investigated by Dr French (*Trans. Opt. Soc.* **21** 1919-20 (136)). In practice ratings are chosen for coincidence range-finding who have a low degree of astigmatism. This can be corrected, if necessary, by fitting a correcting lens to the ocular of the rangefinder.

Mr W. M. Brett: When making tests with the Snellens test types, using the radial fan chart, the patient under test being seated at 6 metres' distance from the chart and the best correction being found for the meridians at right angles to each other, meridians at 45° are not equally distinct, indicating the presence of irregular astigmatism.

Messrs Hamblin and Winser: We are much indebted to Mr Smith for his detailed mathematical analysis of our observations. The conclusion arrived at by Mr Smith, that the lens of this particular eye is astigmatic and that this astigmatism and its axis vary with the state of accommodation, is supported by the fact that the corneal astigmatism measured by a keratometer is only 1.12 D.CYL. axis 175° , leaving a fairly large amount of astigmatism otherwise unaccounted for. This astigmatic property of the lens seems fairly common.

It is not now possible for us to continue the experiments on this particular eye in order to obtain further series.

EXHIBIT OF THE "DEVIOGRAPH"

By OWEN AVES, F.B.O.A.

Exhibited, 10th November, 1927

THE instrument* to be demonstrated is called the "Devio-graph," and is a modification of the "Myophoriograph," an instrument invented by Mr Esdaile of Newport. Its main purpose is to provide the refractionist with the means of obtaining graphic records of muscular imbalance in the para-central field of vision. Research work in problems of muscular imbalance has shown that, quite apart from those cases where there is paralysis of the external ocular muscles, tonic deviations may vary considerably when the eyes are orientated towards different zones of the field of vision.

Hitherto investigations of muscular imbalance, of the tonic variety, have been usually carried out in the principal position, that is to say, the fixation object being situated centrally in front of the face, both in the horizontal and vertical medians. A general survey of different workers' observations in this field of study indicates that varying amounts of deviation have been attributed mainly to the well-known connection between *convergence* and *accommodation*, and where anomalies have appeared scant attention seems to have been paid to their explanation.

The inventor considers that such investigation has been neglected owing to the many mechanical difficulties presented to the refractionist when endeavouring to measure deviations in any other than the central position. The "Devio-graph" endeavours to remove these difficulties by providing a quick and accurate routine technique so that a large amount of clinical material may be available for the further solution of problems affecting the binocular vision.

DESCRIPTION OF APPARATUS

The fixation object consists of a small target made up of eight white circles arranged so as to form a rectangle. This target is carried in a sliding box-like cage containing a magnifying eyepiece and plane mirror. The optical system is designed so that an observer, looking centrally through the eyepiece, perceives a magnified virtual image of the target 35 cm. in front of the eye. The target carrier is carried in a slide provided with a slit at a suitable height to enable the observer to place the carrier in front of either eye at will. A suitable chin-rest and temple-supports give provision for fixing the head so that the eyes are maintained horizontally and in the median line of the slit. Horizontal adjustment is ensured by the provision

* For a fuller description of the instrument see *Dioptric Bulletin*, 29 1927 (439).

of an additional plane mirror placed at the back of the carrier; it can be swung in front of the slit when required for this adjustment.

At a distance of 35 cm. in front of the slit is a desk carrying a hooded shade together with a black silk copying sheet underneath which an ordinary copying carbon is inserted together with the chart. In the base of the instrument a subdued illuminating device provides illumination for the target and sufficient illumination for the white pointed stylus which is operated by the subject under examination. Both the copying desk and the target slide carrier are provided with suitable millimetre scales enabling the operator to move the chart from side to side to distances corresponding to the pupillary distances between the two eyes. The instrument is constructed on a base sufficiently rigid to provide a stable optical bench for the target, eyes, and copying desk.

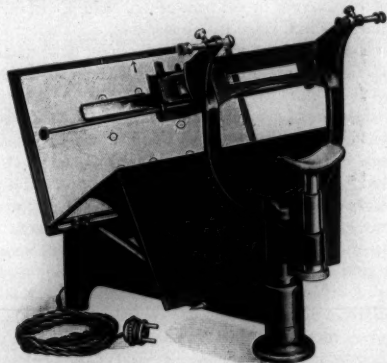


Fig. 1. "Deviograph" with hood lowered for adjustment of chart and carbon copying paper

TECHNIQUE OF THE INSTRUMENT

The chart having been inserted, the patient is directed to place the forehead on the front of the instrument in such a position that the eyes are close to the slit, and in a horizontal position when viewed through the long plane mirror. The head is then fixed in that position by means of the chin-rest and temple-supports, and the plane mirror is swung vertically out of view. The target is then adjusted by the patient, centrally in front of the left eye, and he is instructed to mark on the black desk a small sign or the letter *R* in the middle of each of the eight circles. This operation being completed, the target is moved over in front of the right eye and the chart moved a corresponding distance to the right also. The patient then repeats the operation with the stylus, marking a slightly different sign or the letter *L*. (To ensure accuracy of marking, a cross may be used for the first operation, and

a dot for the second.) The patient is then released and the chart indicates quantitatively and qualitatively the deviations in eight para-central positions of the binocular field.

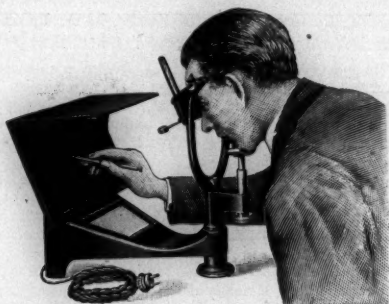


Fig. 2. Patient in position for graphing with right eye whilst looking at target with left eye

NOTES ON CHART READING

(a) The desk being placed 35 cm. in front of the eyes and any error being doubled by the two separate readings, it follows that a deviation of 7 mm. between two corresponding points represents one prism dioptre of deviation.

(b) Coincidence of corresponding points represents orthophoria in that zone.

(c) In lateral deviations, reversal of the signs made by the right and left eye indicates esophoria, whilst non-reversal indicates exophoria.

(d) Hyperphoria is indicated by vertical displacement of corresponding points and is indicated as right hyperphoria or left hyperphoria, according to the nature of the upper sign which has been made by the patient's non-fixing eye.

NATURE OF CHARTS

Observation of a number of charts shows that perfect orthophoria is comparatively rare, and that regular lateral deviations are most common. Hyperphoria is much more common than is usually suspected. The term "regular" prefixed to a muscular defect is used where all the corresponding points have a corresponding amount of deviation. Routine practice with the "Deviograph" shows however that many subjects have irregular anomalies, one of the commonest being that the exophoria decreases as the eyes are moved from an upper to a lower position. A less common observation is that the lateral deviation may be greater on one side than on the other. Still another anomaly, and a fairly common one, is that the hyperphoria may vary from side to side, particularly in the upward direction, and sometimes is so marked that a hyperphoria on one side may become a hypophoria on the other side.

REVIEWS

Applied Refraction. By HOMER ERASTUS SMITH, M.D. Pp. ix + 131; 9 Figs. (Edinburgh: E. & S. Livingstone, 1927.) Price 15s. net. Postage 6d.

The one hundred and twenty pages of this book, written in a condensed and forceful manner, contain a wide range of matters of interest to the refractionist. The author offers advice, based upon his own extensive experience, on the technique governing the detection and correction of refractive errors and includes helpful observations on the lay-out of the test room, the comfort of the patient, the psychological effects of these, the cosmetic effects of spectacles, the manipulation of frames, etc.

The author points out in his preface that the book is not written for the beginner in refraction work. To the refractionist who possesses a sound groundwork in the science of his subject and who has had sufficient experience to equip himself with balance and judgment, the book will be found stimulating and well worth serious attention. The succession of somewhat dogmatic assertions made by the author he will be prepared to read and weigh calmly in the light of his own experience. One can imagine the less experienced student and refractionist, however, being seriously disturbed and frequently confused by these forceful statements, which often have to do with controversial matters. Thus "if one prefers to work with the astigmatic chart, the one here illustrated is by far the most sensitive and delicate test." And again "my own procedure is that any patient who has evidence of eyestrain (note this distinction) be he seven or seventy, shall always be refracted under full cycloplegia."

The author rightly draws attention to the necessity of allowing for the effective power of lenses and the modifications introduced into this important quantity by lens form and thickness and changes in position, but his manner of description does not tend to clarify the subject, so widely misunderstood. Indeed, in certain places misleading statements are made. Thus a rule given on p. 59 for determining the effective power of a lens is merely the ordinary formula for equivalent or "true" power of a thick lens; the *effective* power involves the lens position. Again on p. 88, "a biconvex lens of plus 6.00 diopters does not have a focal length of 16.66 centimetres unless made infinitely thin."

Statements such as those on pp. 58 and 60 to the effect that "the choice cannot be left to the optician, for he will usually take the path of least resistance and make up for your patient an unwearable lens..." and "shall we let him (the optician) work his own sweet will and crab an act which we have worked up to perfection?" are illustrative of a superior attitude still sometimes found amongst medical men and distinctly unfair to the many opticians who are at least equally capable in the domain of lens dispensing. Happily there are not many instances of this attitude in this country.

On the whole the book is well written and tastefully set up and provides a definitely useful contribution to the art of ocular refraction which will be welcomed by the many experienced refractionists, medical and otherwise, who are striving towards still higher efficiency in their work.

H. H. E.

Simple Methods of Surveying from Air Photographs (Professional Papers of the Air Survey Committee, No. 3). By Lieut. M. HOTINE, R.E. Pp. 71; 23 Figs, 2 Plates, and 1 Map. (London: H.M. Stationery Office, 1927.) Price 3s. 6d. net.

The important problems raised by the development of aerial photography as a method of surveying are receiving close attention in all the principal countries of the world. The field is wide and all the problems are difficult because so many diverse factors enter into

each of them. A change in factors which to the lay mind may appear trivial, for example a change in the scale of the map required, may in reality affect the whole of the operations profoundly. Of outstanding importance are the questions of cost, and these depend largely on the actual flying time, and on the quality of the flying. Thus the publication of a reasonably full account of the results of trials over different kinds of country is of the greatest value to all interested in such work. The paper under review contains such an account of a trial survey carried out near Arundel in 1926. A method of plotting offering distinct advantages owing to its simplicity and rapidity was to be employed, and the varying character of the country included in the selected area ensured a searching test. The results obtained are decidedly encouraging for medium and small scale work, such as will generally be required from aerial surveys. The paper gives full details of the routine adopted, contains advice to field workers on a very large number of points some of which might easily be overlooked, sets forth clearly the assumptions made in any part of the work and the reasons for their adoption (as well as the reasons against the acceptance of other assumptions), and includes a drawing based on the survey, which is ingeniously prepared so as to enable a comparison to be made with the corresponding portion of the Ordnance Survey Map. The paper altogether is of such importance that no one who is interested in such work, from whatever point of view, should be without a copy.

T. S.

Penrose's Annual: The Process Year Book and Review of the Graphic Arts, Vol. xxx. Edited by Wm. Gamble, F.R.P.S., F.O.S. Pp. xvi + 160. (London: Percy Lund, Humphries & Co., Ltd., 1928.) Price 8s. net.

The present volume contains the following articles: "A Review of the Year's Process Work," "The Work of the Private Presses: The Golden Cockerel Press," "Exhibition Décor and the Printer," "Arts and Advertisement," "If Art Galleries sent Invitations to Printers," "The Adventurous Trend of Modern Advertising Design," "Advertisement Typesetting: The New 'Fourth Art' in Publicity," "An Analysis of Foreign Types and Some Reasons for their Popularity," "Lettering as a Craft," "A New Tendency in Poster Production," "How Printing Journals could be Improved," "The Growing Appreciation for Colour," "The Misuse of Colour," "A Short Historical Account of Woodcuts and Wood Engraving," "The Beginnings of Half-tone," "Some Thoughts on Printers' Representatives," "Machine Composition and its Problems," "Novel Possibilities in the Collotype Process," "Printing Papers Magnified," "Paper Problems in Offset Printing," "Printing Direct on Metal Plates by Projection," "A New Method of Producing Blocks," "Some Recent Developments in Photogravure," "The Offset Principle in Photogravure," "The Beginning of Rotogravure Printing in America," "Photometric Methods," "Panchromatic Plates," "A New Collodion Method," "A Note on Wet-Collodion Developer," "A Plea for Better Photogravure Etching," "The Advantages of Photo-lithography," "The Better the Original, the Better the Block," "Simple Combination Printing with an Enlarging Lantern," "What is the Other Fellow Doing?," "Random Shots at Printers wanting more Business," and "The Editor's Note Book." The volume contains numerous illustrations representative of process work during the year 1927, in addition to plates illustrating the articles. The firm of Messrs Penrose & Co., Ltd., with which this Annual has been associated from its commencement, has been acquired by Messrs Hunters, Ltd., and will now be run under the title of Messrs Hunter-Penrose, Ltd. It is pleasing to learn that Mr Gamble, who has retired from the firm, will continue to edit the Annual.

ABSTRACTS OF PATENT SPECIFICATIONS

268771*. Zeiss, C.

SIGHT-TESTING APPARATUS

Two arms, carrying respectively a magnifier and an illuminating device, are pivoted upon an axis upon the line of which the pupil of either eye of the subject can be brought.

269231. Kelly, A.

PHOTOMETRIC APPARATUS.

In testing electric lamps for intrinsic brightness, they are viewed through one or more coloured screens of such density that the glowing filament is just visible.

269435. Berthelsen, C. F.

LAMPS.

The source of light is arranged at the common focus of a parabolic reflector and a plano-convex lens of very short focal length, which is surrounded by a ring of slight curvature. The direct rays which, owing to spherical aberration, are not rendered parallel by the lens are brought into parallelism by a lens closing the end of the parabolic reflector.

269658. Baird, J. L.

TELEVISION APPARATUS.

The object is traversed by a spot of light of high intensity, which would be injurious to the object if it were allowed to come to rest. Traversing is effected by a spiral series of lenses, or by a pair of moving prisms placed at right angles to each other.

269718. Connolly, T. F.

THEODOLITES.

Both graduated circles are read by a single compound microscope, the objective of which forms the images upon a diaphragm which is itself provided with a vernier reading against both scales.

269834. Baird, J. L.

TELEVISION APPARATUS.

Synchronism between the sending and receiving stations is maintained by the use of an independent control image, which may be employed to operate a speed indicator at the receiver. A series of light impulses in conjunction with a rotating disc may also be employed.

270119. Barr and Stroud, Ltd., and another.

RANGEFINDERS.

The part of the instrument receiving undeflected rays is provided with a reciprocal scale of ranges, in relation to which the object is viewed, thus allowing the true azimuthal direction to be read off directly.

270251*. Wernicke and Co.

BEVELLING LENSES.

The bevelled edges of flat or convex, round or oval lenses are shaped by a grinding disc of a suitable profile. The work is mounted upon a swinging frame, and the grinding wheel and the work are free to move at right angles to the plane of movement of the work holder. Special work-guiding mechanism is thus dispensed with.

270516. Eckschlagler, H.

REFLECTORS.

A reflector for flood lighting consists of a rear concave portion merging by successive gradations, first into a conical portion and then into a convex portion of considerable spread. As applied to vehicle lamps, the reflector has an intermediate substantially cylindrical portion.

R. J. T.

* Specification published before acceptance.